EVIDENCE FOR FEIGENBAUM UNIVERSALITY IN STOCK INDICES BEHAVIOR

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Abstract

Time series comparison between S&P500 data in 1997 and 1998 has been conducted. Phase trajectories and the corresponding density histograms were built. It turned out that the ratio of distances between neighboring regions of maximum density of the phase trajectories is determined by the Feigenbaum constant $\boldsymbol{\alpha}_{\mathbf{F}}$ = 2.5029… The latter characterizes splittings of phase trajectory in a series of period-doubling bifurcations in the way from order to chaos for many nonlinear dynamical systems. Thus, $\boldsymbol{\alpha}_{\mathbf{F}}$ can be used in forecasting stock indices behavior.

1. The idea of nonlinearity more and more penetrates into the economic analysis and the capital markets theory [1], calling the methods of fractal geometry [2] and the theory of deterministic chaos [3[, [4] for financial research. The results obtained this way allow us to reveal tendencies which are unable to be explained by the Efficient Market Hypothesis: long memory of the capital market, delay in response to the new information, non-integer dimensions of most experimental data series.

Our method is based on the following remarkable observation: if a solution (as a numerical series) of some unknown equation obeys to a set of special conditions, then we can predict the future behavior of this solution with no finding the explicit equation. Particularly, it is true for the whole class of non-linear differential equations, whose solutions evolve in accordance with the Feigenbaum scenario [5]. These solutions must satisfy soft enough special conditions: the negative value of the Schwartz derivative and the existence of only one maximum for the corresponding Poincaré map [6]. The main quantitative patterns remain the same for any dynamical system of this class.

2. Consider the stock market as a mechanical system consisting of many elements – stocks. Any dynamical system can be characterized by two variables: some dynamical parameter and its first time derivative. These variables are called the generalized coordinate **q** and the generalized speed **q'**, respectively. As the generalized coordinate of the element of the stock market, the price of the corresponding stock is usually taken. As the generalized coordinate of the whole stock market, **Q**, the sum of the all stock prices weighed with the corresponding emitter capitalization, **m**, and normalized to the total capitalization of the market, **M**, is taken. The variable analogous to **Q** is already used in practice. In the United States, for example, the S&P Composite plays such a role. The full list of correspondence between the stock market terms and mechanical system variables in the frame of our method is represented in **Table 1.**

Mechanical system variable	**Stock market term**	**Symbol**
Generalized coordinate	Stock price	q
Generalized speed	Stock price change per unit of time	$q' = \frac{dq}{dt} = q(t) - q(t-1)$
Mass of an element of the system	Capitalization of an emitter	m
Coordinate of the center of mass	Stock index	$Q = \frac{\sum_{i=1}^{N} m_i \times q_i}{\sum_{i=1}^{N} m_i}$
Speed of the center of mass	Stock index change	$Q' = \frac{dQ}{dt}$
Total mass of the system	Total capitalization of N emitters	$\sum_{i=1}^{N} m_i$

In the stock market, the *phase trajectory* is just the dependence of the stock price change on the price itself: the dependence of the type **q'(q)**. Every drastic change in the behavior of dynamical system corresponds to some qualitative change (for example, splitting) of the phase trajectory [4], [7]. In particular, such behavior of the trajectory can be observed in nonlinear dynamical systems evolving from order to chaos according to the Feigenbaum scenario [5]: if the governing parameter of the system changes smoothly in a certain interval, then the system undergoes a series of period-doubling bifurcations - transitions from limit point to the 2-cycle, from the 2-cycle to the 4-cycle, etc. These bifurcations possess universality: in infinity, the ratio of any two successive splittings scale tends to the Feigenbaum constant, $\boldsymbol{\alpha}_{\mathbf{F}}$ = 2.5029….

3. To demonstrate abilities of our phase trajectory method, we use the S&P 500, which plays the role of the generalized coordinate **Q** in the US stock market. The data for analysis - daily S&P 500 closing values - were taken from the web-site [9]. As a result for these two years, our data set consists of 496 experimental values: 247 values in 1997 and 249 values in 1998. We found that in the middle of 1997 (June-July), the phase trajectory of the stock market underwent qualitative changes, having passed from one region of the phase space into another with the scale of the space trajectory splitting increasing $\boldsymbol{\alpha}_{\mathbf{F}}$ times. Thus, drastic change in the stock market took place not in October of 1997, but much earlier – in June or July, which might have been an indicator of pre-crisis phenomena. The new S&P 500 maximum at **Q =** 1100 could have been predicted as early as in the summer of 1997 !

References

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