Trading Rules and Trading Volume

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ABSTRACT. Investors who use fixed trading rules conditioned on stock characteristics must rebalance their portfolios regularly because characteristics fluctuate over time. This paper presents evidence that such rule-driven rebalancing accounts for a substantial portion of trading volume in stock markets. I develop a statistical model in which trading volume depends on fluctuations in stock characteristics and on the heterogeneity and persistence over time of investors' trading rules. To calibrate the model, I estimate these heterogeneity and persistence parameters from mutual fund holdings data. Applied to observed changes in stock characteristics, the calibrated model suggests that rule-driven trading generates about 25% of NYSE/AMEX trading volume. Panel regression results are consistent with the magnitudes obtained in the calibration: Stocks for which rule-driven trading volume is predicted to be higher tend to have higher actual trading volume, controlling for other determinants of volume and using instrumental variables to account for the endogeneity of stock characteristics.

Keywords: Trading Volume; Investor Behavior; Mutual Funds

JEL classification: G10, G20

1. Introduction

Why do financial market participants trade? Turnover on the NYSE is currently running at about 100% per year.[1] Trading volume in foreign exchange markets amounts to about $1.2 trillion per day.[2] Yet, it is not well understood why investors undertake these trades. Theoretically, there are a variety of potential explanations. Information asymmetry might play a role, although it does not easily explain trade on its own. As shown in Akerlof (1970) and Milgrom and Stokey (1982), rational uninformed agents would recognize the information advantage of informed traders, leading to a no-trade situation. Trading volume can arise, though, if market incompleteness forces some investors to trade in response to uninsured idiosyncratic shocks. Such trade for hedging reasons would provide "noise" in the order flow that could sustain profitable trading on private information.[3] Alternatively, investors might trade because they "agree to disagree" and interpret information differently.[4] It seems plausible that some of the observed trading activity is indeed driven by these motives, but it is difficult to establish how much. Trading volume could be zero in complete markets and infinity in incomplete markets with zero transaction costs and continuous information flow.[5]

There is yet little empirical evidence that one could bring to bear on this question. Recent work has uncovered some of the determinants of investors' propensity to trade—for example, Grinblatt et al. (1995), Grinblatt and Keloharju (2001), Cohen et al. (2002), and Sias (2002) show that investors' decisions to trade are related to past returns, and tax considerations, cash-flow news, and herding; Barber and Odean (2001) and Glaser and Weber (2003) link individual trading decisions to overconfidence—but these findings do not allow a quantitative assessment of how much different trading motives contribute to observed trading volume.

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[1] Defined as the number of shares traded over the number of shares outstanding.
[2] See Bank of International Settlements (2002). For comparison, annual U.S. GDP is about $10 trillion in 2002.
[3] See, e.g., Grossman and Stiglitz (1980), Kyle (1985), Wang (1994), and He and Wang (1995). Dow and Gorton (1997) argue that noise trading might arise because portfolio managers churn their portfolios to appear skilled to their principals. Brunnermeier (2001) provides a more complete survey of the theoretical literature on trading volume.
[4] Harrison and Kreps (1978), Varian (1989), Harris and Raviv (1993), Kandel and Pearson (1995), Odean (1998), and Scheinkman and Xiong (2003) are models in which investors trade because of differences in beliefs.
[5] See Lo, Mamaysky, and Wang (2001) for a calibration of a model with fixed transaction costs.

In this paper, I also study micro-level trading patterns, but I do so in a novel framework that allows judging their effect on trading volume. Specifically, I explore empirically the role of *trading rules* in generating trade in stock markets. Trading rules are defined as a fixed mapping from a set of publicly observable asset characteristics—e.g., historical price patterns, accounting ratios, or analyst forecasts—into buy and sell decisions. The following strategy, described by a mutual fund manager, would fit this definition well:

> *"First we start out with a universe of 9,700 to 9,800 stocks, then narrow our focus to companies with a market cap minimum of $172 million. Then we screen for stocks that have price-to-sales ratios of below 1.5* […]*, whose annual earnings are higher now than in the previous year* […]*, companies that have positive relative strength in their share price over the past three, six and 12 months.* […] *Most of the time it's all new faces when we rebalance."* [6]

The link to trading volume is as follows. When investors use such fixed strategies to form their portfolios, trading needs arise—as mentioned by the fund manager above—because assets change their characteristics over time, forcing investors to replace assets whose characteristics no longer fit. This trading activity is neither driven by liquidity shocks, nor induced by private information. My goal in this paper is to estimate empirically how much trading volume is caused in this way. The basic insight that my analysis builds on is that if investors follow trading rules, stocks with more pronounced changes in characteristics should experience larger trading volume.

Trading rules seem to be ubiquitous in financial markets and can come in different disguises. One can distinguish two broad classes: Trading rules that arise from delegated portfolio management, and trading rules that serve as forecasting heuristics. Trading rules in the first category are often referred to as investment styles. Pension funds, mutual funds, and hedge funds tend to focus their investments on assets with certain common attributes, e.g. small-cap or growth stocks, as documented in Brown and Goetzmann (1997), Fung and Hsieh (1997), Chan et al. (2002) and Wermers (2002). Theoretically, these styles could address heterogeneous hedging needs in the investor population, as in

[6] From an interview with Neil J. Hennessy, manager of the Hennessy Cornerstone Growth Fund on www.fundemail.com, April 24, 2002.

Lo and Wang (2001b) and Mamaysky and Spiegel (2002), but it is perhaps more plausible that product differentiation and agency problems play an important role. For example, differentiation in styles could be a way for funds to cater to those retail investors that happen to believe in the superiority of certain investment styles [Massa (2000), Gabaix and Laibson (2003)]. Barberis and Shleifer (2003) suggest that styles may simplify asset allocation decisions and facilitate the performance evaluation of money managers.

Trading rules in the second category—forecasting heuristics—include technical analysis and quantitative trading strategies, for example. These strategies commonly involve buying and selling conditional on some fixed set of statistical patterns [see, e.g., Lo and Wang 2001a]. A simple example is momentum investing, which calls for buying stocks that recently outperformed the market. Other technical trading rules may be more complex, but they are nonetheless driven by the same basic principle that trading decisions are deterministically linked to some historical price patterns. Also, traders with a more fundamentally oriented approach might use fixed rules of thumb based on valuation ratios or growth rate forecasts. One can view these trading rules as heuristics that investors use to form opinions about future stock returns based on public information. Faced with limited information processing capacity, they might use simple models, conditioned on a limited number of variables and reevaluated only infrequently, perhaps along the lines of Mullainathan (2002) and Hong and Stein (2003). With data snooping, and heterogeneity in modeling techniques, horizons, and sophistication, it seems plausible that different investors could come up with different models, leading to heterogeneity in trading rules.

To assess how prevalent rule-driven trading is, and how much trading volume it generates, I begin by setting up a statistical model that captures two central aspects of rule-driven trading. First, investors form portfolios focused on stocks with certain characteristics. The flipside of this focus is that there is cross-investor dispersion in the average characteristics of their portfolios, which I refer to as *location dispersion*. The narrower investors' focus, the greater must be the location dispersion. Second, when stock characteristics change over time, rule-bound investors trade back to their previous portfolio composition. Therefore, if many investors follow trading rules, the average investor exhibits high *location persistence*. Calibrated with estimates for location dispersion and persistence, the model predicts how much rule-driven trading volume should arise from a given change in a stock's characteristics—the higher location dispersion and persistence, the higher the trading volume. The model can accommodate multiple characteristics, taking into account their correlation.

A key feature of the model is that the location dispersion and persistence parameters can be estimated from investor portfolio holdings data. This allows me to calibrate the model without fitting free parameters to match trading volume statistics. Its trading volume predictions can thus be subjected to empirical tests. To obtain estimates for location dispersion and persistence, I use data on mutual fund holdings. Since there is only little evidence in the prior literature as to the stock attributes that play important roles in trading rules, I use a broad range of stock characteristics, including price momentum, valuation ratios, growth forecasts, profitability, risk measures, firm size, and others.

The first important result is that mutual funds exhibit considerable location dispersion with respect to many of these characteristics. It is most pronounced for forecasted earnings growth, dividend yield, size, and leverage. Furthermore, I find that many mutual funds tend to keep the location of their portfolio fixed over time, i.e. there is location persistence. To give a specific example, some mutual funds focus on high growth stocks and others on low growth stocks, and many undertake trades to maintain their particular growth focus over time. Under some assumptions, the estimate for this location persistence can be interpreted as the fraction of investors who follow fixed trading rules. During the sample period 1984 to 2000, this proportion fluctuates between 30% and 50%.

Based on these estimates for location dispersion and persistence, and using the observed changes in stock characteristics at annual frequency, the calibrated model suggests that rule-driven trading accounts for a total turnover of about 18% per year for NYSE/AMEX stocks from 1984 to 2001, which is roughly 25% of the actual trading volume during this period. Interestingly, it seems that momentum characteristics have the greatest impact on rule-driven trading volume, despite only moderate location dispersion, because they are highly volatile—much more so than firm size, for example. This is a hint that a significant part of rule-driven trading might arise from technical trading rules. Many technical characteristics are likely to be correlated with price momentum. For example, a stock that switches from being a loser to being a winner is also likely to break through trendlines and experience changes in other technical indicators.

Implicitly, the calibration assumes that mutual funds' trading rules are approximately representative for the trading rules prevailing in the general investor population. Therefore, to check the calibration results, I run cross-sectional regressions with a panel of NYSE/AMEX stocks. Consistent with the model, the results show that one percent higher predicted rule-driven turnover, based on each stock's observed change in characteristics over annual observation intervals, leads to approximately one percent higher actual turnover. In these regressions, I employ instrumental variables to address the fact that changes in many stock characteristics are endogenous, and that other unobservable determinants of trading volume—such as information flow, for example—might be correlated with changes in stock characteristics. Overall, the quantitative predictions of the calibrated model are borne out in the data.

In summary, my results suggest that the use of trading rules is widespread. Rule-based trading is not induced by liquidity needs or private information, but it appears to account for a substantial portion of trading volume in the stock market. To be clear, it is important to note that my rule-driven trading volume estimates capture only the trades that are undertaken to maintain a portfolio consistent with a fixed rule, but not the trades that result from switching of rules. Rule-switching, either at the level of institutional investors' clients, as in Barberis and Shleifer (2003), or by investors themselves could certainly lead to additional trading volume. However, unlike fixed rule trades, which have a clear and tractable relation to changes in stock characteristics, it is less transparent what might drive investors' decisions to switch rules. Hence my focus on fixed trading rules.

The rest of the paper is organized as follows. Section 2 presents the statistical model. In Section 3, I calibrate the model using mutual fund portfolio holdings data. Section 4 presents the empirical tests using a panel of NYSE and AMEX stocks. Section 5 concludes.

2. The Relationship Between Trading Rules and Trading Volume

Suppose that some investors follow trading rules. When stock characteristics change, these investors trade to maintain portfolios consistent with their trading rule. How much trading volume would that generate for the typical stock? The statistical model that I develop in this section provides the necessary structure to answer this question. With this structure in place, one can describe investors' portfolio holdings along many characteristics dimensions with a relatively small number of estimable parameters, and the link between trading rules and trading volume can be made explicit. To focus on the essential aspects, the model abstracts from issues of pricing and from investors' deeper motivations for following trading rules. For the purposes of quantifying rule-driven trading volume, it suffices to describe in statistical terms how stocks are distributed across investor portfolios, and how this distribution evolves over time. As long this statistical description is correct, my trading volume results must hold.

2.1 Assets, characteristics, and trading rules

The model is set in discrete time, $t = 0, 1, \ldots, T$. There is a continuum of stocks, indexed by $i \in \mathbb{R}$, and with total mass (market capitalization) normalized to one. Each stock i is endowed with a vector $c_{it} \equiv [c_1 \ldots c_K]'_{it}$ of K time-varying characteristics, e.g. dividend yield, past returns, profitability, etc. For each stock i, c_{it} follows a mean-zero mean-reverting stochastic process (e.g., a stationary vector autoregressive process) that has reached a steady state such that the cross-sectional distribution is time-invariant and given by $c_{it} \sim N[0, \Sigma_c]$. Thus, individual stocks' characteristics change over time, but their cross-sectional distribution in the population of stocks, denoted $f_c(c)$, remains the same. [7] To follow the development of the model, it is helpful to think of c_{it} as being just a scalar of one

[7] It seems plausible that what matters most for investors' choices is the value of c_{it} relative to other stocks in the same time-period. Accordingly, one can simply think of c_{it} as representing standard normal scores rather than the absolute value of stock characteristics. In this case, the time-invariance of the cross-sectional distribution holds by definition. In the empirical analysis, all characteristics are transformed to obey a standard normal cross-sectional distribution.

characteristic—say, profitability—and Σ_c (and the other covariance matrices that I define below) simply as variances, although all results apply for multivariate c_{it}.

There exists further a continuum of investors, also with total mass (capital) equal to one, and indexed by $j \in \mathbb{R}$, holding the entire supply of stocks. Trading rules, as defined in this paper, have two properties. First, they involve some focus on stocks with certain characteristics, and second, these characteristics preferences are fixed over time. To capture the first property in a tractable way, I assume that the distribution of stock characteristics within investor j's portfolio is normal with mean m_{jt} and covariance Ω_c. I refer to m_{jt} as the *location* of investor j's portfolio and to Ω_c as the *within-portfolio dispersion*.[8] Taking the profitability example, the trading rule might prescribe a narrow focus on stocks with high profits, but allowing for some variation (Ω_c) around a target for profitability (m_{jt}). With respect to the second property, it would be unrealistic to assume that all investors in the stock market always follow a fixed trading rule. Instead, I assume that with probability γ, investor j sticks to her location from the previous period, with probability (1-γ) she chooses a new one. I refer to γ as the *location persistence*. The following analysis focuses on trading volume only among the set of investors with fixed rules—that is, those who target $m_{jt} = m_{jt-1}$. Therefore, until we get to the empirical work, it is not necessary to be more specific on how the other investors choose m_{jt}. In summary:

Assumption: *Investor j chooses stocks i such that* $c_{ijt} \sim \mathrm{N}[m_{jt}, \Omega_c]$. *With probability* γ *investor j chooses* $m_{jt} = m_{jt-1}$.

Note, however, that in my empirical work below, I estimate the parameters Ω_c and γ from portfolio holdings data. At this point, I do not mean to suggest a priori that their magnitude should be large or small. The only critical components of the assumption are normality and homoskedasticity (identical Ω_c for all investors), which permit to aggregate across investors in a tractable fashion. Of course, investors' portfolios might be more heterogeneous in reality. It seems plausible that some investors

[8] Note, though, that Ω_c is a covariance matrix, not a scalar, but for ease of reference, I often simply refer to it as "dispersion" or "variance".

might seek a portfolio focused on one characteristic, say, momentum, and neglecting others, while other investors might neglect momentum and focus on valuation ratios, for example. But even so, my simplified setting could still provide a useful approximation. For example, even if investors differ in their within-portfolio dispersion, the within-portfolio distribution of the average investor—while not exactly normal—might still be well approximated by a normal distribution, unless the difference in within-portfolio dispersions is too large. It is the within-portfolio distribution of the average investor with given m_j that drives the model's predictions. Similarly, it is possible that individual investors form portfolios that are non-normal, e.g. uniform with different variances, while the portfolio of the average investor is still approximately normal. Ultimately, it will be left to the empirical tests later in the paper to determine whether the model approximates investors' portfolios well enough to produce useful predictions.

These few assumptions about investors' investment policies are all that is needed to derive trading volume implications. For this purpose, it is useful to analyze the joint density function $f(m, c)$, which describes how the mass of market equity is distributed across stocks—distinguished by c_i—and across investors—distinguished by m_j. Too see how it can be constructed, note that the marginal distribution $f_c(c)$ is given exogenously. Next, based on the above assumption, we also know the conditional distribution $f(c|m)$. It is simply the distribution of c_{it} within the portfolio of an investor j with $m_{jt} = m$, that is, $c|m \sim N[m, \Omega_c]$. Finally, the remaining marginal distribution $f_m(m)$, which describes the distribution of m_{jt} across investors, is pinned down by the market clearing condition: The density of stocks with characteristics c demanded by investors in aggregate must always equal the density $f_c(c)$ in the stock supply. Appendix A shows that this implies

$$m_{jt} \sim N[0, \Sigma_m], \text{ where } \Sigma_m = \Sigma_c - \Omega_c \text{[9]} . \quad (1)$$

[9] This also implies that m_{jt} obeys the same distribution for investors that stick to their old trading rules and those investors that choose a new m_{jt}. Among the first group $m_{jt} \sim N[0, \Sigma_m]$, hence the same distribution also has to apply to the second group, irrespective of the stochastic nature of their new choice, as it has to be $m_{jt} \sim N[0, \Sigma_m]$ in aggregate, too.

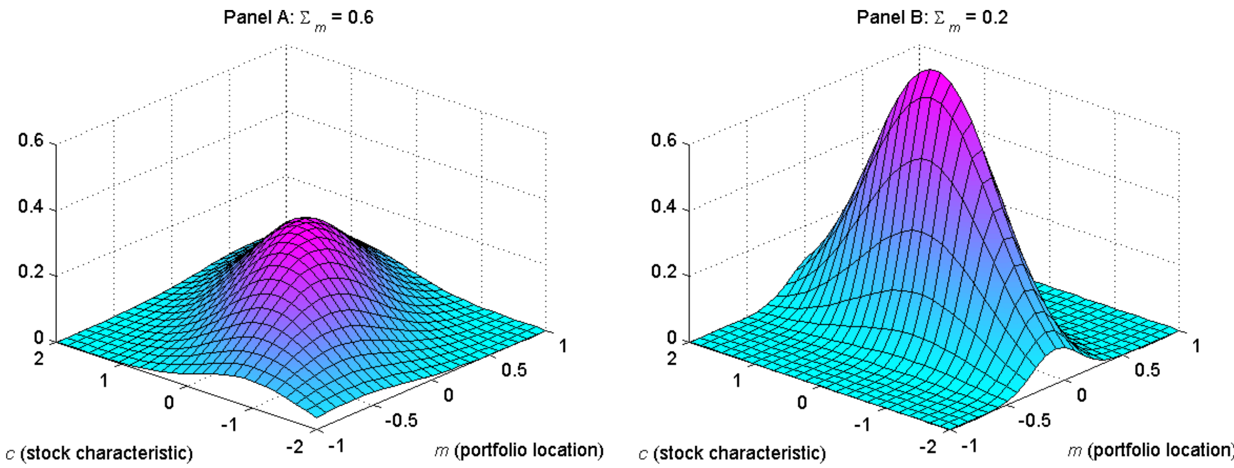

Figure 1
Surface plots of the joint distribution of stock characteristics and portfolio locations. Examples with high (Panel A) and low (Panel B) location dispersion.

The intuition behind this result is most easily seen when there is only one stock characteristic. For example, if investors aim for portfolios with low Ω_c (e.g., narrowly focused on stocks with similar profitability), then *location dispersion* Σ_m must be high (i.e., some investors must focus on low profitability and some on high profitability stocks), otherwise the market would not clear. Note that this must be true under very general conditions. It does not depend on how prices are set, or on the deeper motivations that lead investors to follow trading rules. If investors hold focused portfolios—whatever the reason for it—there has to be dispersion in their portfolio location for the market to clear.

Since $f_m(m)$ and $f(c|m)$ are multivariate normal, $f(m, c) = f(c|m)f_m(m)$ is multivariate normal, too (see Appendix B for its parameters). To illustrate, Figure 1 presents two examples of $f(m, c)$ for the single characteristic case. For concreteness, let c be profitability, as before. In both graphs, $\Sigma_c = 1$. Panel A depicts a situation in which investors aim for narrowly focused portfolios (low Ω_c). Loosely speaking, investor j's portfolio, $f(c|\ m_j)$, can be thought of as a "cut" through $f(m, c)$ parallel to the c-axis at $m = m_j$. For example, investors with m_j = -1 hold stocks with c mostly between 0 and –2, i.e. low profitability. The investors at the opposite end with $m_j = 1$ are invested in high profitability stocks. As can be seen, the low within-portfolio dispersion is associated with high location dispersion between

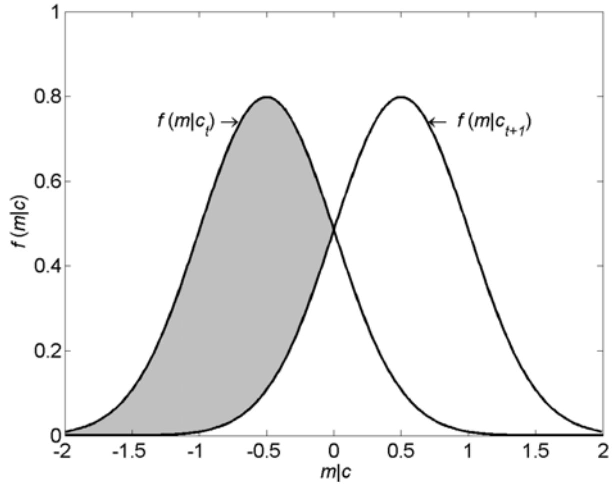

Figure 2
Trading volume in response to characteristics changes

investors (high Σ_m). In contrast, in Panel B, investors hold portfolios that are more diversified with respect to c (high Ω_c). As a result, there is only little dispersion in m_j across investors (low Σ_m)—their portfolios are almost identical in terms of their characteristics composition.

2.2 Changes in characteristics and trading volume

The shape of $f(m, c)$ is time-invariant. Individual stocks change their characteristics, and a fraction $(1-\gamma)$ of all investors change their portfolio location each period, but the overall distribution remains unchanged. Based on $f(m, c)$, we can now back out how much trading volume is caused each period by trading rules. To see the effects most transparently, suppose for a moment that $\gamma = 1$ and so every investor j has a constant portfolio location m_j. In that case, it is easy to see that if stock i's characteristics change from c_{it} to c_{it+1}, its owner clientele must change, too. Since investors have constant portfolio location, we can express these owner clienteles in terms of the owners' m_j—that is, the conditional distributions $f(m \mid c_{it})$ and $f(m \mid c_{it+1})$. Figure 2 illustrates this point with an example, again for scalar c. Put informally, the depicted distributions correspond to "cuts" parallel to the m-axis at $c = c_{it}$ and $c = c_{it+1}$ through $f(m, c)$ (recall Figure 1). For example, when a stock has low profitability in t (low c_{it}), most of its market equity is held by investors that aim for low profitability stocks (low

m_j). If the stock's profitability increases from t to t+1, its owner clientele shifts towards investors that aim for high profits. Since investors' portfolio locations are unchanged over time, this change in $f(m|\ c)$ can only happen through trade. The dark shaded area represents the fraction of the stock's shares that is turned over from low to high m_j investors.

When $\gamma < 1$, the conditional distributions are the same as with $\gamma = 1$. However, only a fraction γ of the investor population now maintains $m_{jt+1} = m_{jt.}$ Since my focus in this paper is exclusively on the trading volume caused by fixed, deterministic trading rules, I count only the trades among investors with fixed m_j as rule-driven trading volume. Since they account for a fraction γ of the total mass of investors, this is simply the dark shaded area in Figure 2 times the location persistence parameter γ. The following definition states this more formally:

Definition: *Rule-driven trading volume (turnover) of a stock experiencing a change in characteristics* $\Delta c_{it} \equiv c_{it+1} - c_{it}$ *is*

$$\tau_{it} \equiv \gamma \int_{m \in S} f\left(m \mid c_{it}\right) - f\left(m \mid c_{it+1}\right) \mathrm{d}m\,, \tag{2}$$

where $S \equiv \left\{m : f\left(m|c_{it}\right) > f\left(m|c_{it+1}\right)\right\}$

The integral in Eq. (2) is simply the continuous equivalent of the standard definition of turnover, summing shares sold across all investors, standardized by total shares outstanding.

It is worth emphasizing that we do not need to know anything about trading costs and other frictions to quantify rule-driven trading volume here. If we observe that investors have high location persistence (high γ), and that they hold focused portfolios and hence location dispersion is high (high Σ_m), this state is only sustainable if investors trade. Otherwise, with mean-reverting characteristics, their portfolios would eventually converge to an identical characteristics distribution as $t \to \infty$ and location dispersion would go to zero. For the same reason, it does not matter that some characteristics—in particular those based on price, e.g. price/sales—might be endogenous with respect to trading volume. Eq. (2) is simply a consequence of market clearing and must hold even if trading itself influences characteristics.[10] Integration of Eq. (2) leads to the following result (see Appendix C):

Proposition: *In each period t, the rule-driven trading volume (turnover),* τ_{it}*, of stock i depends on its change in characteristics* Δc_{it} *as follows:*

$$\tau_{it} = 2\gamma\left(\Phi\left(\tfrac{1}{2} d_{it}\right) - \tfrac{1}{2}\right), \tag{3}$$

where

$$d_{it} = \sqrt{\Delta c_{it}' \Sigma_c^{-1} \Sigma_m \Omega_m^{-1} \Sigma_m \Sigma_c^{-1} \Delta c_{it}}\ , \tag{4}$$

and $\Phi(.)$ *denotes the standard normal CDF and* $\Omega_m \equiv \mathrm{Var}[m \mid c]$.

To see the intuition behind this result, note that d_{it} is a distance measure.[11] A stock that travels Δc_{it} through c-space is expected to travel along the vector $\Delta m_{it} = \Sigma_m \Sigma_c^{-1} \Delta c_{it}$ through m-space, i.e. across investor portfolios ordered by their location m. The length of this vector in m-space, standardized by Ω_m, is given by d_{it}. The greater d_{it}, the more pronounced is the change in the owner clientele induced by Δc_{it}, and hence the higher the trading volume. Accordingly, τ_{it} is a monotone increasing (nonlinear) function of d_{it}. In short, the proposition provides a way to condense the trading volume implications of changes in characteristics along many (correlated) dimensions into one single number. The essence of this result is that there should be more rule-driven trading volume when (i) investors have greater location dispersion (larger d_{it} for a given Δc_{it}), (ii) their location persistence is higher (higher γ), and (iii), characteristics are more volatile (larger Δc_{it}). The rest of the paper is devoted to quantifying and testing this relationship.

[10] Yet, endogeneity does play an important role in empirical tests of the model predictions, as I explain in more detail in Section 4 of the paper.

[11] More precisely, d_{it} is a Mahalanobis distance [see, e.g., Anderson (2003, p. 80)]. With diagonal Ω_m and unit variances on its diagonal, d_{it} would reduce to the familiar Euclidean distance.

3. Calibration

Calibration of the model requires estimates for the covariance matrices Σ_c, Σ_m, Ω_m, and the location persistence γ. Given these estimates, one can use Eq. (3) to predict stock-by-stock how much rule-driven trading volume should result from a change in characteristics Δc_{it}. Ideally, one would want to estimate these parameters from data on stock holdings of all market participants. Of course, such data sets only exist for some subsets of the investor population. I use data on mutual fund stock holdings, which are the most comprehensive portfolio level data currently available. The calibration exercise is then conducted under the working assumption that mutual funds are approximately representative—in terms of their trading rules—for the general investor population. However, the empirical tests that follow will reveal the accuracy of the predictions obtained in the calibration.

3.1 Identification and estimation of calibration parameters

All parameters are estimated cross-sectionally and are allowed to vary over time. The covariance matrices Σ_{mt} and Σ_{ct} are estimated within each quarter, using value-weights:

$$\hat{\Sigma}_{ct} = \frac{1}{s_t}\sum_{i=1}^{I}\left(c_{it} - \bar{c}_t\right)\left(c_{it} - \bar{c}_t\right)' s_{it} \tag{5}$$

$$\hat{\Sigma}_{mt} = \frac{1}{s_t}\sum_{j=1}^{J}\left(m_{jt} - \bar{c}_t\right)\left(m_{jt} - \bar{c}_t\right)' s_{jt} , \tag{6}$$

where $m_{jt} = \frac{1}{s_j}\sum_i c_{it} s_{ijt}$ is fund j's portfolio location, s_{ijt} is fund j's investment in stock $i, s_{it} = \sum_j s_{ijt}$ denotes the market capitalization of stock i, $s_{jt} = \sum_i s_{ijt}$ is the market value of investor j's portfolio, $s_t = \sum_i \sum_j s_{ijt}$ denotes the value of the market portfolio, and $\bar{c}_t = \sum_i c_{it} s_{it}$ is the value-weighted sample mean of characteristics across all stocks. The estimator of Σ_{mt} is centered at $\bar{c}_t$ rather than $\bar{m}_t$ to take into account that mutual funds in aggregate may have some trading rules bias. If the data covered the entire investor population, it would always be the case that $\bar{m}_t = \bar{c}_t$. Eq.

(6) thus also incorporates the dispersion between mutual funds and other investor groups. This is in analogy to the fact that if the population mean of m_{jt} is known to be $\overline{c}_t$, the maximum likelihood estimator of the population covariance Σ_{mt}, using a sample (mutual funds in this case) with sample mean $\overline{m}_t \neq \overline{c}_t$, is Eq. (6). Finally, given the estimates for Σ_{ct} and Σ_{mt}, and by imposing multivariate normality as in the model, we can estimate the conditional covariance Ω_{mt} as

$$\hat{\Omega}_{mt} = \hat{\Sigma}_{mt} - \hat{\Sigma}_{mt}\hat{\Sigma}_{ct}^{-1}\hat{\Sigma}_{mt} . \tag{7}$$

The identification of the location persistence parameter γ is a bit more intricate. To disentangle investors with fixed trading rules and those without, we need to be more specific about the trading patterns of those without rules. It is useful to consider first the case of one stock characteristic. Let us start by decomposing the location drift of investor j's portfolio from t to t+1, i.e.

$$m_{jt+1} - m_{jt} = \sum_{i=1}^{I}\left(s_{ijt+1}c_{it+1} - s_{ijt}c_{it}\right) , \tag{8}$$

into *active* (*ALD*) and *passive* location drift (*PLD*):[12]

$$m_{jt+1} - m_{jt} = \underbrace{\sum_{i=1}^{I}\left(s_{ijt+1}c_{it+1} - \tilde{s}_{ijt+1}c_{it+1}\right)}_{ALD_{jt+1}} + \underbrace{\sum_{i=1}^{I}\left(\tilde{s}_{ijt+1}c_{it+1} - s_{ijt}c_{it}\right)}_{PLD_{jt+1}} , \tag{9}$$

PLD_{jt+1} denotes the change in m_j that the investor would experience with a pure buy-and-hold policy: the portfolio weights change from s_{ijt} to $\tilde{s}_{ijt+1}$ merely because of price changes between t and t+1. ALD_{jt+1} refers to the change in m_j that the investor achieves by trading from the buy-and-hold weights $\tilde{s}_{ijt+1}$ to the actual weights s_{ijt+1}. The key to estimating γ is that m_{jt+1} depends in different ways on its own lag and on lagged *PLD* for investors that follow trading rules and for those that do not.

Consider first the investors with trading rules. With continuous rebalancing, they would always keep their portfolio location equal to their time-invariant target, which shall be denoted x_j, i.e $m_{jt+1} = m_{jt} = x_j$. More realistically, though, investors might revise their portfolios less frequently and

[12] This terminology borrows from Wermers (2002), who analyzes "style drift" of mutual fund portfolios.

their rebalancing dates need not coincide with the quarter-ends at which m_{jt} is measured. As a result, the observed m_{jt} is only a noisy measure of x_j, i.e. $m_{jt} = x_j + \varepsilon_{jt,}$ with $\mathrm{Cov}(x_j, \varepsilon_{jt}) = 0$. Therefore, $m_{jt+1} = m_{jt} - \varepsilon_{jt} + \varepsilon_{jt+1}$. As will become clear, for identification it is useful to project ε_{jt} onto PLD_{jt}, i.e. $\varepsilon_{jt} = s\ PLD_{jt} + \eta_{jt}$, which yields:

$$m_{jt+1} = m_{jt} + s\ PLD_{jt+1} - s\ PLD_{jt} - \eta_{jt} + \eta_{jt+1}\ . \tag{10}$$

Here, η_{jt+1} is uncorrelated with all the other right hand side variables, but it is important to keep in mind that $\mathrm{Cov}(m_{jt}, \eta_{jt}) > 0$, because $\mathrm{Cov}(m_{jt}, \varepsilon_{jt}) > 0$. Furthermore, $\mathrm{Cov}(PLD_{jt+1}, \eta_{jt},)$ is likely to be nonzero, because η_{jt} is a component of m_{jt}, and, with mean-reverting characteristics, PLD_{jt+1} is correlated with m_{jt}. Consider now investors without fixed trading rules. When they choose new stocks for their portfolios, their choice of characteristics is not related to the portfolio location in the previous period. Hence, the subportfolio of new stocks has random location u_{t+1} with $\mathrm{Cov}(u_{t+1}, m_t) = 0$ and $\mathrm{Cov}(u_{t+1}, PLD_{t+1}) = 0$. However, it need not be the case that they turnover the entire portfolio. The part they hold on to drifts passively towards $m_{jt+1} = m_t + PLD_{t+1}$. On average therefore the following relationship holds:

$$m_{jt+1} = \pi\,(m_{jt} + PLD_{jt}) + (1 - \pi)\ u_{jt}\ , \tag{11}$$

where $0 < \pi < 1$ depends on how strongly the investors turn over their portfolios.

If the probability that investor j is of the first (fixed-rule) type is γ, and the probability for the second type is $(1\text{-}\gamma)$, averaging across all investors yields the following mixture of Eqs. (10) and (11):

$$m_{jt+1} = [\gamma + (1\text{-}\gamma)\ \pi]\ m_{jt} + [s\ \gamma + \pi\,(1\text{-}\gamma)]\ PLD_{jt+1} - [s\ \gamma\,]\ PLD_{jt} + \nu_{jt+1} \tag{12}$$

Eq. (12) prescribes a cross-sectional regression that can be used to identify γ (by subtracting the sum of the second and third coefficients from the first). The intuition behind Eq. (12) is simply that the greater the fraction of investors with fixed rules, the stronger is the persistence in m_{jt}, controlling for current and past passive location drift. Yet, identification is complicated by the fact that the disturbance ν_t inherits the η_t terms from Eq. (10) and thus also the correlation with m_{jt} and PLD_{t+1}. Hence, OLS does not deliver consistent estimates of the coefficients in Eq. (12). To identify γ, I use

$m_{t\text{-}1}$ and ξ_{t+1}, the residual in the projection $PLD_{jt+1} = r\ m_{jt} + \xi_{jt+1}$, as instruments in two-stage least squares (2SLS). Both instruments are correlated with the regressors m_{jt} and PLD_{t+1}, and uncorrelated with the structural disturbance components η_t, η_{t+1}, and u_{t+1}. Hence, Eq. (12) is exactly identified.

Within the framework of the model, the characteristics that feature in rule-bound investors' trading rules should all have identical γ, while other characteristics that are neglected should have zero γ. Of course, this might not be exactly true in reality, because some characteristics may be more popular among rule traders than others—an aspect that is not captured in the model. As an approximation, I use cluster analysis to separate the initial list of candidate stock characteristics into two groups with high and low γ. More specifically, I estimate Eq. (12) cross-sectionally by 2SLS for each characteristic and quarter, take the time-series average these estimates, and feed these sample period averages into a 2-means clustering algorithm. This algorithm allocates characteristics to the two groups such that the average squared deviation from the mean within each group is minimized. I then use only the characteristics in the high-γ cluster to calibrate the model. The characteristics in the low-γ cluster are excluded from the analysis, which effectively amounts to assuming that their γ is zero. Among the high-γ characteristics, I take the cross-sectional average of their γ estimates in each quarter to arrive at a single estimate $\hat{\gamma}_t$ that can be used to calibrate the model.

3.2 Data and summary statistics

The parameters described in the previous section are estimated from mutual fund holdings data extracted from the Thomson Financial Mutual Funds (Spectrum) Database. The database contains quarterly stock-by-stock positions of most U.S. mutual funds, and is explained is explained in great detail in Wermers (1999). I match these quarterly holdings observations with data on stock characteristics from CRSP, COMPUSTAT and I/B/E/S. In principle, one would want to select the stock characteristics that mutual funds use as a basis for their trading rules. Yet, there is little guidance as to which characteristics this would be. Existing empirical work by Brown and Goetzmann (1997), Chan et al. (2002), and Wermers (2002) shows that mutual funds tend to cluster on firm size, measures related to growth or glamour (e.g., book-to-market) and momentum. However, an exhaustive search has not yet been conducted in the literature. For this reason, I consider a wide range of characteristics and let the data speak as to their relevance for mutual funds' trading rules.

My initial list of candidate characteristics is drawn up based on the following principles. First, they should be plausibly related to firm value or risk, motivated by the conjecture that investors might most likely condition their trading rules on such characteristics. Second, to be consistent with my model, the characteristics should be continuous variables. This rules out discrete variables such as analyst recommendations or index membership. Third, the characteristics should have sufficient time-variation. Only relatively volatile characteristics are likely to be sources of rule-driven trading volume. For example, while it might seem plausible that some investors select firms based on R&D expenses, most firms have quite persistent ratios of R&D expenses to sales, making it an unlikely candidate to explain trading volume. The same is true for many other accounting variables.

Based on this reasoning, I select the following characteristics: Momentum (*Ret12*: past 12-month returns; *Ret3*: past 3-month returns; *Mav12*: price minus 12-month moving average price), glamour characteristics (*Ret36*: 36-month return; *S/M*: log of sales/market equity[13]), profitability measures (*E/S*: earnings/sales; *E/A*: earnings/assets), leverage (*L/A*: liabilities/assets; *L/M*: liabilities/market equity), firm liquidity (*CF/L*: cash flow/liabilities; *C/CL*: cash and short-term investments/current liabilities), growth (*FutGr*: I/B/E/S long term growth rate forecast; *EGr*: one-year change in earnings/assets; *SGr*: one-year change in sales/assets), yield (*DY*: dividend yield), risk measures (*Beta*: past 36-month beta on the value-weighted CRSP index; *Volat*: one-year variance of daily returns), and firm size (*Size*: market capitalization).

In the end, any selection of characteristics has to involve some arbitrary element, but the results are not very sensitive to the exact definition of the characteristics set. First, the distance

[13] I use sales/price and not book-to-market to avoid having to eliminate observations with negative book values. Towards the end of the 1990s there is a fairly large number of firms with negative book values.

measure in Eq. (4) does not change much if one adds additional characteristics that are highly correlated with linear combinations of the existing ones. It seems hard to think of other variables that might not be highly correlated with the existing ones. Second, data mining is not an issue here. Adding a variable to the set of characteristics would only produce a higher rule-driven trading volume estimate if mutual funds during the sample period were dispersed in their preferences for this characteristic (high Σ_m), and if they traded to offset location drift with respect to this characteristic (high γ)—that is, if many funds indeed followed trading rules based on this characteristic.

Since the model in Section 2 is based on normally distributed characteristics, I transform all variables to standard normal scores in each quarter. This is consistent with the idea that investors are likely to select stocks based on the relative level rather than the absolute level of characteristics. More precisely, because all statistics in the calibration are value-weighted, I force the characteristics to obey a standard normal distribution in value-weighted terms. Since I/B/E/S long-term growth forecasts are not available before 1982 and a lag of two years is required to compute the variables in Eq. (12), the sample period starts in the first quarter of 1984. It ends in the fourth quarter of 2000. The sample only includes NYSE and AMEX stocks, because Nasdaq trading volume figures include interdealer trading, which makes them incompatible with those from NYSE and AMEX [see Atkins and Dyl (1997)].[14] Finally, in any given quarter, I include only stocks that have a valid observation for each of the 18 candidate characteristics.

Changes in characteristics (Δc_{it}) and passive location drift (PLD_{jt}) are calculated each quarter over overlapping annual intervals. There are two reasons for this choice of interval: First, many of the stock characteristics are based on accounting data that change only once per year. For these characteristics, shorter observation intervals would not make much sense. Second, while most of the mutual fund holdings data is updated each quarter, the data for some funds are reported at semi-annual or even annual frequency only [see, also, Wermers (1999)]. Using annual intervals thus avoids many

[14] In principle, one could also estimate the calibration parameters including the data on Nasdaq stocks, and then focus on NYSE/AMEX stocks in the empirical tests. This leads to similar results.

complications arising from stale holdings information. To be consistent with the annual measurement interval for Δc_{it}, I average the quarterly $\hat{\gamma}_t$, $\hat{\Sigma}_m$ and $\hat{\Sigma}_c$ estimates over the four quarters contemporaneous with Δc_{it}.

Table 1 shows some summary statistics for the sample of stocks and mutual funds used in this study. The statistics reported in this table are time-averaged cross-sectional estimates. All statistics, percentiles as well as correlations, are calculated using value-weighting, using stock or fund market capitalization as weights. As shown in the bottom line, the data requirements leave, on average, 866 stocks in the sample, which is about 42% of the number of NYSE/AMEX stocks on CRSP. Since these stocks tend to be large stocks, they account for 69% of NYSE/AMEX market capitalization. The average number of mutual funds in the Thomson Financial database that hold at least one of these stocks with valid data is 1695, and the median fund holds about 56 NYSE/AMEX stocks, while the mean is 84.

Univariate summary statistics for c_{it} would of course be inherently uninteresting, because all characteristics are transformed to standard normal scores. However, statistics on the distribution of portfolio location (m_{jt}) are a useful diagnostic. The calibration uses a joint normality assumption, so it would be helpful if the distribution of m_{jt} were at least approximately normal. Also, it would be interesting to know whether mutual funds in aggregate have strong preferences for certain characteristics. As the first block of rows shows, the mean and median fund seems to have some preference for stocks with low profitability (*E*/*S*, *E*/*A*), high forecasted growth (*FutGr*), low dividend yield (*DY*), high risk (*Beta* and *Volat*), and smaller *Size*. Yet, relative to the cross-sectional standard deviation of c_{it}, which is equal to one, the means and medians are still relatively close to zero. The 5^{th} and 95^{th} percentile values show that the cross-sectional distribution of m_{jt} is reasonably symmetric for most characteristics, except perhaps for *Volat* and *Size* which exhibit considerable skewness. Overall, there is reason to expect that the normality assumption for m_{jt} should provide an adequate approximation.

Table 1
Summary statistics on standardized stock characteristics and mutual funds' portfolio locations

This table shows summary statistics using stocks that have a valid observation on each of the 18 candidate characteristics in a given quarter. The sample period runs from the first quarter of 1984 to the last quarter of 2000. Medians, percentiles, and correlations are calculated cross-sectionally each quarter, using value-weights, and are then averaged over time. For stock characteristics (c_{it}), the unit of observation is a stock. For portfolio location (m_{jt}), the unit of observation is a fund, and m_{jt} refers to the fund *j*'s vector of mean characteristics, computed by value-weighting (using portfolio weights) the c_{it} vectors of all stocks in funds *j*'s portfolio. The first block of rows shows statistics on the cross-sectional distribution of m_{jt}. The second block shows the sample correlation matrices for the elements of c_{it} (upper triangular part) and m_{jt} (lower triangular part) that correspond to the sample estimates of Σ_c and Σ_m, which are defined in Section 3.1. of the text.

	Ret12	*Ret3*	*Mav12*	*Ret36*	*S/M*	*E/S*	*E/A*	*L/A*	*L/M*	*CF/L*	*C/CL*	*FutGr*	*EGr*	*SGr*	*DY*	*Beta*	*Volat*	*Size*
									portfolio location (m_j)									
Mean	0.04	0.02	0.04	0.01	0.07	-0.11	-0.12	0.01	0.06	-0.09	0.03	0.09	0.00	0.03	-0.13	0.16	0.17	-0.22
Median	0.01	0.02	0.02	-0.01	0.07	-0.09	-0.11	0.04	0.06	-0.08	0.00	0.07	0.01	0.03	-0.14	0.15	0.14	-0.13
5^{th} pct.	-0.57	-0.47	-0.51	-0.71	-0.67	-0.67	-0.71	-0.60	-0.75	-0.65	-0.41	-0.80	-0.46	-0.32	-1.12	-0.47	-0.53	-1.13
95^{th} pct.	0.74	0.53	0.67	0.83	0.83	0.37	0.44	0.50	0.91	0.46	0.56	1.05	0.41	0.39	0.89	0.78	0.98	0.34
											-- correlation among elements of c_i --							
Ret12		0.50	0.82	0.57	-0.26	0.00	0.02	0.04	-0.22	-0.01	0.04	0.14	0.01	0.05	-0.11	0.04	0.03	0.16
Ret3	0.55		0.74	0.28	-0.13	0.01	0.02	0.02	-0.11	0.00	0.01	0.04	0.01	0.02	-0.02	0.02	0.02	0.09
Mav12	0.88	0.75		0.63	-0.24	-0.01	0.03	0.02	-0.22	0.00	0.03	0.14	0.00	0.03	-0.14	0.05	0.04	0.12
Ret36	0.72	0.34	0.48		-0.39	0.16	0.23	0.01	-0.37	0.14	0.09	0.33	0.27	0.14	-0.23	0.11	0.08	0.22
S/M	-0.46	-0.20	-0.42	-0.66		-0.64	-0.50	0.37	0.72	-0.48	-0.32	-0.44	-0.11	-0.04	0.26	-0.05	-0.01	-0.23
E/S	0.07	0.01	0.02	0.29	-0.59		0.74	-0.27	-0.38	0.63	0.23	0.06	0.40	0.05	0.10	-0.13	-0.23	0.24
E/A	0.19	0.08	0.18	0.46	-0.59	0.73		-0.47	-0.68	0.87	0.30	0.31	0.51	0.08	-0.13	0.01	-0.07	0.21
L/A	-0.15	-0.04	-0.17	-0.25	0.54	-0.23	-0.49		0.71	-0.72	-0.47	-0.29	-0.06	-0.03	0.23	-0.05	-0.10	0.13
L/M	-0.46	-0.20	-0.45	-0.66	0.86	-0.37	-0.70	0.75		-0.73	-0.43	-0.62	-0.18	-0.08	0.44	-0.15	-0.17	-0.11
CF/L	0.18	0.06	0.18	0.40	-0.61	0.60	0.89	-0.75	-0.78		0.40	0.28	0.40	0.10	-0.14	0.00	-0.02	0.15
C/CL	0.25	0.10	0.26	0.39	-0.51	0.14	0.40	-0.68	-0.67	0.57		0.27	0.09	0.03	-0.23	0.08	0.15	-0.02
FutGr	0.41	0.16	0.41	0.60	-0.64	0.03	0.45	-0.61	-0.85	0.55	0.66		0.11	0.04	-0.75	0.45	0.48	-0.01
EGr	0.11	0.03	0.08	0.38	-0.18	0.38	0.52	-0.12	-0.24	0.43	0.17	0.21		0.33	-0.06	0.03	-0.02	0.06
SGr	0.11	0.07	0.09	0.16	-0.05	0.03	0.06	0.02	-0.06	0.06	0.00	0.06	0.38		0.01	-0.03	-0.01	0.02
DY	-0.35	-0.13	-0.37	-0.48	0.46	0.16	-0.24	0.55	0.68	-0.38	-0.65	-0.91	-0.16	-0.04		-0.48	-0.58	0.20
Beta	0.23	0.09	0.24	0.33	-0.21	-0.26	0.15	-0.33	-0.45	0.24	0.47	0.71	0.14	-0.01	-0.76		0.51	-0.11
Volat	0.24	0.09	0.26	0.31	-0.22	-0.37	0.00	-0.38	-0.45	0.16	0.56	0.74	0.08	0.02	-0.86	0.79		-0.31
Size	0.13	0.08	0.08	0.19	-0.23	0.40	0.36	0.19	-0.11	0.21	-0.11	-0.12	0.07	0.00	0.32	-0.18	-0.42	
						-- correlation among elements of m_j --												
#stocks:	866			#stocks per fund:	mean		84											
#funds:	1695				median		56											

The matrix in the second block shows cross-sectional correlation estimates for c_i (upper triangular part) and for m_j (lower triangular part). These correlation matrices are derived from $\hat{\Sigma}_{ct}$ and $\hat{\Sigma}_{mt}$, respectively, and are then averaged over time. If mutual funds held all stocks in the market, and absent estimation issues, both correlation matrices, since value-weighted, would have to be identical. As the table shows, most of the estimated correlations for c_i and m_j are similar. It is also apparent that many characteristics are highly correlated. For example, not surprisingly, *Ret12* and *Ret36* are positively correlated (0.57). Perhaps more interestingly, *DY* and *FutGr* have strong negative correlation (-0.75). Many of the other correlations have magnitudes around 0.50 or –0.50. This highlights the importance of accounting appropriately for correlation among stock characteristics in the derivation of rule-driven trading volume in Section 2.

3.3 Determining the set of relevant characteristics

As the first step to calibrate the model, I determine which characteristics play a relevant role in investors' trading rules in the sense that many investors choose to keep constant their portfolio location with respect to this characteristic. Following the clustering algorithm described in section 3.1, I form two groups of characteristics with low and high estimates for location persistence γ, respectively. As it turns out, the results are clear-cut. The low-γ cluster contains three characteristics: *Ret3*, *EGr*, and *SGr*, with γ-estimates of 0.16, 0.10, and 0.04, respectively. The γ-estimates for the other characteristics in the high-γ cluster are all much higher and lie between 0.30 and 0.50, as I show in more detail below. Evidently, only few investors follow fixed trading rules based on *Ret3*, *EGr*, and *SGr*, and changes in these characteristics are therefore unlikely to produce much rule-driven trading volume. Hence, for the rest of the paper, I work only with the characteristics in the high-γ cluster.

3.4 Unidimensional calibration results

To provide some insight as to the characteristics that feature most prominently in investors' trading rules, I start with a unidimensional calibration exercise, calibrating the model one characteristic at a time. In interpreting these univariate findings it is important, though, to keep in mind that characteristics are correlated, and thus the results may partly reflect commonality across characteristics.

Table 2 presents estimates of the calibration parameters. Recall that according to the model developed in Section 2, the amount of rule-driven trading volume is related positively location dispersion (Σ_m), location persistence (γ), and the magnitude of characteristics changes ($|\Delta c_{it}|$) (All characteristics are transformed to standard normal scores, so Σ_c is simply equal to one). Since we examine one characteristic at a time here, all parameters are scalars. They are estimated quarter-by-quarter, and the table shows their time-series averages, along with the associated autocorrelation-consistent standard error in parentheses. The first column presents estimates for σ_m, the square root of Σ_m. To judge the magnitudes, it is useful to compare them to the dispersion that one would expect if funds did not have any intention to construct a focused portfolio. In this case, any variation in m_{jt} would just be a product of chance, and σ_m would be equal to the standard error of the mean of c in a randomly sampled portfolio. For the median fund with 55 stocks, this standard error would be about 0.14, assuming that stocks have identical value-weights. As the table shows, the observed location dispersion between funds is much higher. Funds exhibit particularly strong dispersion with respect to *FutGr* (0.58), *DY* (0.60), and *Size* (0.49). *Volat*, *L/M*, *Ret36* and *S/M* also exihibit large values for Σ_m. There is less location dispersion for the firm liquidity variables and for profitability and for most of the purely accounting-based variables like *E/S* or *CF/L*. The standard errors of $\hat{\sigma}_m$ are generally small, at most 0.02. Evidently, location dispersion among mutual funds tends to be quite stable over time.

Table 2
Unidimensional calibration parameters and results

Each quarter, I calculate the portfolio location (m_{jt}) for each mutual fund, and I estimate the value-weighted standard deviation (σ_{mt}) of m_{jt} between funds (the square root of Σ_{mt}), as described in Section 3.1, but separately for each characteristic. The location persistence (γ_t) is calculated by running cross-sectional 2SLS regressions across funds of m_{jt+1} on m_{jt} and passive location drift, as described in Section 3.1. Characteristics volatility is measured each quarter by taking the value-weighted mean of the absolute values of Δc_{it} over (overlapping) annual intervals across all stocks in the sample. Predicted rule-driven turnover is calculated for each stock in each quarter according to Eq. (4), using the estimates of Σ_{mt}, Ω_{mt} and γ_t, and the observed Δc_{it} over overlapping annual intervals. The individual predicted turnover figures are then aggregated across stocks, weighted by market capitalization. The table shows the time-series average of these statistics, along with Newey-West autocorrelation-consistent standard errors in parentheses. The sample period runs from the first quarter of 1984 to the fourth quarter of 2000. Only NYSE and AMEX stocks are used.

Stock characteristic		Location Dispersion Between Funds (σ_m)	Location persistence (γ)	Characteristics Volatility ($\lvert\Delta c_{it}\rvert$)	Predicted Turnover (in %) (τ)
Momentum	*Ret12*	0.40	0.50	1.04	9.18
		(0.01)	(0.03)	(0.01)	(0.65)
	Mav12	0.37	0.44	1.03	7.32
		(0.01)	(0.02)	(0.01)	(0.45)
Glamour	*Ret36*	0.47	0.32	0.63	4.41
		(0.02)	(0.04)	(0.01)	(0.75)
	S/M	0.45	0.46	0.27	2.62
		(0.01)	(0.02)	(0.01)	(0.19)
Profitability	*E/S*	0.34	0.30	0.48	2.23
		(0.01)	(0.02)	(0.02)	(0.19)
	E/A	0.36	0.33	0.50	2.85
		(0.01)	(0.04)	(0.01)	(0.34)
Leverage	*L/A*	0.34	0.41	0.28	1.63
		(0.00)	(0.03)	(0.01)	(0.10)
	L/M	0.49	0.45	0.26	2.61
		(0.01)	(0.04)	(0.01)	(0.24)
Firm Liquidity	*CF/L*	0.34	0.37	0.44	2.45
		(0.01)	(0.03)	(0.01)	(0.26)
	C/CL	0.30	0.39	0.40	2.02
		(0.01)	(0.02)	(0.02)	(0.17)
Growth	*FutGr*	0.58	0.47	0.27	3.66
		(0.02)	(0.02)	(0.01)	(0.25)
Yield	*DY*	0.60	0.47	0.26	3.89
		(0.02)	(0.02)	(0.01)	(0.14)
Risk	*Beta*	0.40	0.30	0.48	2.77
		(0.02)	(0.03)	(0.04)	(0.28)
	Volat	0.48	0.37	0.53	4.62
		(0.02)	(0.03)	(0.02)	(0.30)
Firm Size	*Size*	0.49	0.49	0.16	2.02
		(0.01)	(0.04)	(0.01)	(0.24)

The second column presents estimates for the location persistence γ, obtained via the regression (12) with 2SLS. These are the numbers that provided the input to the clustering algorithm that attributed the characteristics listed in the table to the high γ-group. Recall that the higher the estimate for γ, the more likely it is that a given fund follows a fixed trading rule on this characteristic. The results show that $\hat{\gamma}$ varies from 0.30 and 0.50. About two thirds of the estimates are within two standard errors from the overall mean of 0.41. Hence, the assumption that γ is constant across these characteristics, which is used in the multidimensional calibration below, is at least approximately in line with the data. The standard errors are relatively small (between 0.02 and 0.04), and thus the results are not clouded much by noise. Taken together, these estimates suggest that on average about 40% of mutual funds stick to their trading rules in a given period.

The third column gives an impression as to the typical magnitudes of $|\Delta c_{it}|$. It reveals substantial differences between characteristics. The momentum characteristics *Ret12* and *Mav12* tend to be the most volatile characteristics, with mean $|\Delta c_{it}|$ of 1.04 and 1.03 per year, respectively. To judge these magnitudes, recall that the cross-sectional standard deviation of c_{it} is standardized to one. Thus, the typical stock changes its momentum characteristic by about one cross-sectional standard deviation per year. In contrast, the two leverage measures *L/A* (0.28) and *L/M* (0.26), and also *FutGr* (0.27), *DY* (0.26), *S/M* (0.27) and especially *Size* (0.16) are much less volatile.

The differences in characteristics volatility turn out to be quite important for the relative magnitudes of (predicted) rule-driven trading volume (τ_{it}) shown in the final column. It is calculated by applying Eqs. (3) and (4) at the individual stock level, using the univariate estimates for Σ_{mt} and γ_t along with Δc_{it}. These numbers are then averaged (using value-weights) across stocks and then across time. Interestingly, momentum-related trading rules seem to be the most important generators of rule-driven trading volume, with predicted annual turnover of 9.18% (std.err. 0.65%) and 7.32% (std.err. 0.45%) per year for *Ret12* and *Mav12*, respectively. The volatility of these returns-based characteristics implies that it requires a lot of trading to maintain a momentum or a contrarian portfolio.

Conversely, *Size*, which scores high both on Σ_m and $\hat{\gamma}$, and thus seems to be one of the major sources of clustering among mutual funds, produces comparatively low τ_{it} of only 2.02% (std.err. 0.24%). *Size* simply does not change enough over time to cause a lot of rule-driven trading, at least not in value-weighted terms. Other characteristics that appear to contribute substantially to rule-driven trading volume are the risk characteristics, especially *Volat* with 4.62% (std.err. 0.30%), long-run past returns *Ret36* with 4.41% (std.err. 0.75) and also *DY* and *FutGr*. It seems worth emphasizing at this point that τ_{it} is a prediction, based only on the observed changes in characteristics for each stock and the location dispersion and location persistence parameters estimated from mutual fund holdings ($\hat{\Sigma}_{mt}$ and $\hat{\gamma}_t$). In other words, the trading volume numbers shown in Table 2 are an estimate of how much trade is needed to maintain the observed location dispersion and location persistence. They are not a decomposition of actual trading volume or fitted to actual trading volume.

3.5 Multidimensional calibration results

Since many of the characteristics in Table 2 are highly correlated, the unidimensional results do not allow drawing conclusions about the overall volume of rule-driven trading. In this section, I use the multidimensional version of the model to quantify the aggregate rule-driven trading volume. Now the sample covariance matrices $\hat{\Sigma}_{mt}$, $\hat{\Sigma}_{ct}$ and $\hat{\Omega}_{mt}$ include the covariances of all 15 stock characteristics, and $\hat{\gamma}_t$ is the average of location persistence estimates at time t across all characteristics (i.e., those in the high-γ cluster). Using these estimates along with stock-by-stock observations on Δc_{it}, Eq. (4) yields a prediction for rule-driven trading volume, τ_{it}, for each stock within each (overlapping) annual estimation interval. The first row of Table 4 shows some descriptive statistics of the cross-sectional distribution of τ_{it}. All statistics are computed with value-weighted observations, quarter by quarter, and are then averaged in time. The time-series standard error is reported in parentheses.

Table 3
Multidimensional calibration results and comparison with actual trading volume. Cross-sectional distribution.

Each quarter, I calculate m_{jt} as the value-weighted average of characteristics within fund j's portfolio, and I estimate value-weighted covariance (Σ_{mt}) of m_{jt} between funds. The covariance of characteristics (Σ_{ct}) is estimated from each cross-section of c_{it}. Ω_{mt} is estimated from $\hat{\Sigma}_{mt}$ and $\hat{\Sigma}_{ct}$ as described in section 3.1. Location persistence (γ_t) is estimated first separately for each characteristic by running cross-sectional 2SLS regressions of m_{jt+1} on m_{jt} and passive location drift, as described in Section 3.1. These characteristic-specific estimates are then averaged in each quarter, yielding an estimate for γ_t. Predicted rule-driven turnover (τ_{it}) is calculated for each stock in each quarter according to Eq. (4), using the estimates for Σ_{mt}, Ω_{mt}, and γ_t, and the observed Δc_{it} over overlapping annual intervals. Each quarter, I calculate value-weighted cross-sectional statistics of τ_{it}. The table shows the time-series average of these cross-sectional statistics, with Newey-West autocorrelation-consistent standard errors in parentheses. The sample period runs from the first quarter of 1984 to the fourth quarter of 2000. Only NYSE and AMEX stocks are used. The second row shows the same cross-sectional statistics for total actual trading volume.

	Mean	Median	Std.dev.	5^{th} pct.	95^{th} pct.
Predicted Rule-Driven Turnover (τ_{it})	0.175	0.167	0.069	0.076	0.301
	(0.008)	(0.008)	(0.004)	(0.005)	(0.014)
Actual NYSE/AMEX Turnover	0.715	0.587	0.492	0.283	1.561
	(0.047)	(0.038)	(0.059)	(0.021)	(0.123)

As the table shows, the mean predicted rule-driven trading volume is about 17.5% per year. Note that this is considerably less than what one would get by simply adding up the univariate turnover predictions across different characteristics in Table 2. This is because the distance measure in Eq. (4) properly accounts for the fact that many of these characteristics are strongly correlated. For comparison, the second row shows the mean actual turnover for the same sample of NYSE and AMEX stocks. It is defined as the number of shares traded in a given year, as reported on CRSP, divided by the number of shares outstanding, value-weighted across stocks. This value-weighted mean turnover is equal to aggregate turnover, defined as the fraction of aggregate market equity that is traded in a given year. Over the sample period 1984 to 2000, aggregate turnover amounts to 71.5%. Compared with this number, the calibration results thus suggest that about 25% of aggregate trading volume is caused by rule-driven trading. Considering that one would not expect trading rules to constitute the only reason for trade, this is a substantial amount. Of course, the calibration rests on some strong assumptions. Therefore, in the next section, I subject these results to a careful empirical test.

The percentiles shown in the table reveal that rule-driven turnover tends to be less skewed than total turnover. Evidently, there are some stocks in each period that have quite extreme actual turnover well in excess of 100 percent. As a matter of principle, rule-driven turnover—at least if based on annual rebalancing—can never exceed 100%. Hence, there is no chance that rule-driven trading could ever explain much of these large volume observations. Rule-drive trading is more likely to explain a part of the "base level" of trading volume rather than instances of extremely active trading. Unusually high information flow or trading frenzies are perhaps more likely to be the driving forces behind the more extreme trading volume observations.

This point is also underscored by Figure 3, which shows a four-quarter moving average of aggregate NYSE/AMEX turnover (Panel A) and the time-series of aggregate rule-driven turnover (Panel B), calculated in the same way as in Table 3 over overlapping annual windows. As can be seen in the figure, actual turnover exhibits more pronounced low frequency movements than rule-driven turnover does. The two series are positively correlated (the correlation is not statistically significant after adjusting for autocorrelation, though), and both have their minimum around 1992, but actual turnover almost doubled in the post-1992 years, while rule-driven turnover only increased by about 50% during the same period. In absolute magnitudes, the differences in time-series variation are even bigger. Hence, rule-driven trading cannot help much to explain the pronounced peaks in trading volume during the boom years before the crash in 1987, and during the bull market at the end of the 1990s. This mirrors the cross-sectional results in Table 3, where rule-driven trading volume cannot contribute much to explaining the highest actual trading volume observations.

To some, the lack of a strong time trend in rule-driven turnover in Panel B may appear surprising. Based on casual empiricism, one might have expected that the increased popularity of "investment styles" and the increased product differentiation and specialization in the institutional money management industry would have resulted in an increased volume of rule-driven trading in the

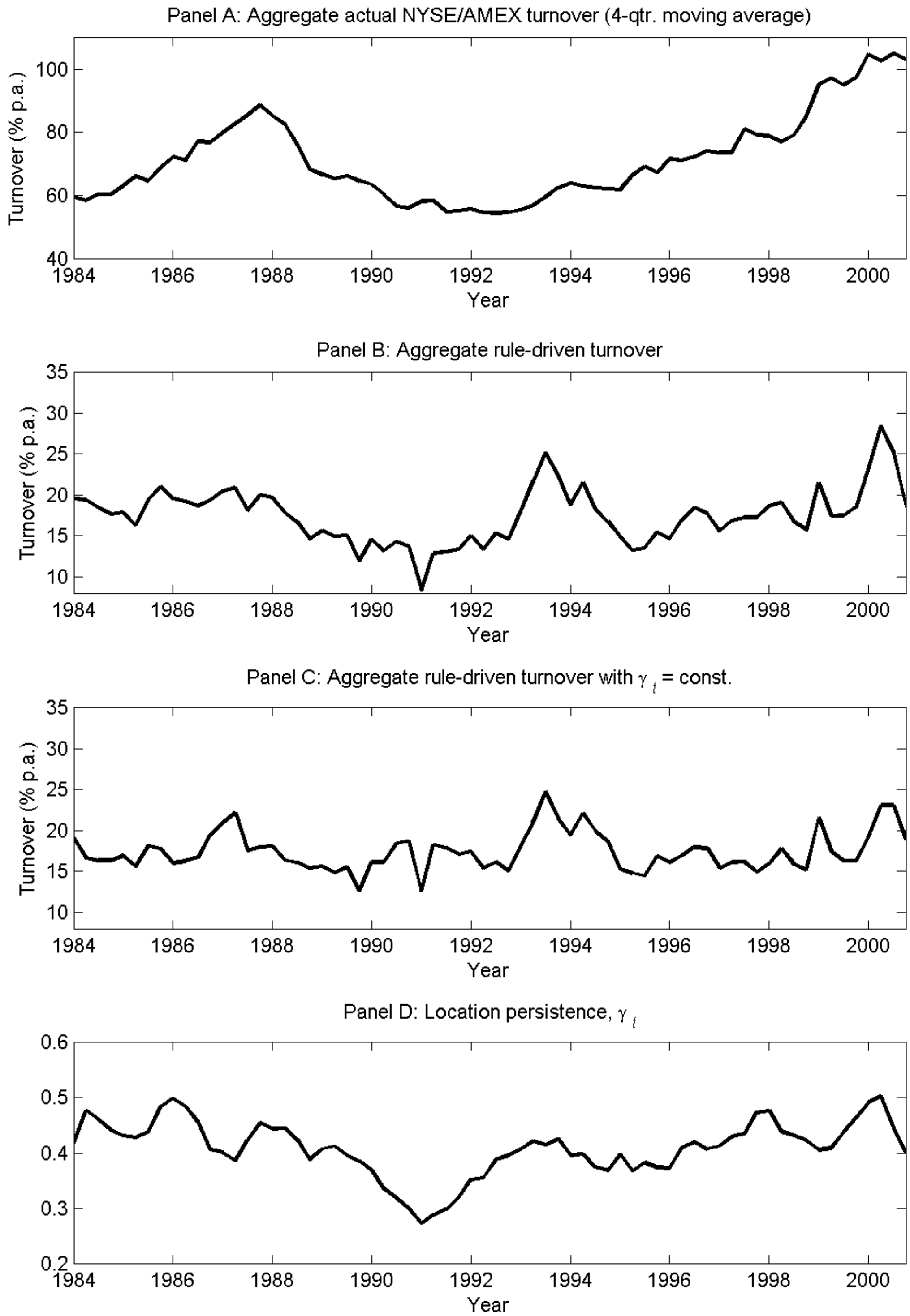

Figure 3
Value-weighted actual NYSE/AMEX turnover and predicted rule-driven turnover, 1983-2000.

later part of the sample. However, there are no clear time-trends in location dispersion among mutual funds. *Size* and *Beta* are the exceptions. With respect to *Size*, σ_m (the square root of the diagonal element of Σ_m corresponding to *Size*) has slowly and steadily increased from about 0.48 in 1984 to 0.58 at the end of the period, indicating an increasing tendency of mutual funds to have a *Size* focus. Of course, as the previous analysis showed, *Size* does not contribute much to rule-driven trading volume, because its volatility is so low. For *Beta* the time-trend looks similar. In fact, it is perhaps not so clear that heterogeneity among funds should have gone up much over time. Trading rules in the form of value, growth, relative-strength (momentum) and contrarian investing, and technical analysis are by no means new ideas. It is quite possible—and the data does not suggest otherwise—that such trading rules were prevalent in the earlier part of the sample, too.

Even though time-variation in rule-driven trading volume is moderate, it would be interesting to know its source. Time-variation can either be due to changing characteristics volatility, changing Σ_m, or due to variation in the fraction of fixed-rule traders (γ_t). Panel C shows the time-series of predicted rule-driven trading volume when γ_t is held constant at its time-series mean. Compared with Panel B, the series still shows a similar degree of volatility at higher frequencies, part of which may be estimation noise. But it lacks some of the lower frequency movements. Evidently, most of the lower frequency variation in rule-driven turnover comes from variation in $\hat{\gamma}_t$. There appear to be times when investors cling more strongly to their trading rules than in others. As can be seen in Panel D, there is some persistent low frequency variation in $\hat{\gamma}_t$. Unfortunately, the persistence in these variables and in actual turnover also imply that it is difficult to draw statistically precise inference on the relationship between rule-driven and actual trading volume from these relatively short aggregate time-series. For this reason, the empirical tests in the next section look at the cross-section of trading volume instead.

4. Relationship with Actual Trading Volume

The calibration analysis above implicitly assumes that all investors exhibit approximately the same rule-trading behavior (in terms of location dispersion and location persistence) as mutual funds do. Moreover, the model also assumes a lot of structure (e.g., normal distributions, equal Ω_c across investors, etc.), which may or may not be appropriate. To check whether its predictions are borne out in the data, this section explores the relationship of predicted rule-driven trading with actual empirically observed trading volume. Everything else equal, stocks for which predicted rule-driven trading volume is higher should have higher actual trading volume. Since the calibration of the model did not involve fitting free parameters to match actual trading volume, such tests are feasible. The tests use panel regressions on NYSE/AMEX stocks. The section concludes with some robustness checks.

4.1 Identification and estimation

Neglecting other influences on trading volume for a moment, the trading rules model would lead to a simple structural model:

$$Turn_{it} = \beta_{0t} + \beta_{11}\tau_{it} + \beta_{12}\tau_{it-1} + \varepsilon_{it} \ , \qquad (13)$$

where $Turn_{it}$ is the turnover of stock i in period t, and τ_{it} is the predicted rule-driven trading volume (as a function of Δc_{it}). I also include the one-year lag $\tau_{it\text{-}1}$, because fixed-rule investors might trade only at recurring rebalancing points, e.g. quarterly or annual. In this case, their trades may take place some time after characteristics have changed. The trading rules model predicts that $\beta_{11} + \beta_{12} = 1$. Of course, this model omits other variables that should also cause trading volume. Since the omitted variables may be correlated with τ_{it} and $\tau_{it\text{-}1}$, OLS estimation of Eq. (13) would not deliver consistent estimates of β_{11} and β_{12}. Furthermore, τ_{it} and $\tau_{it\text{-}1}$ are endogenous: Some of their variation is driven by price changes—that is, by individual stock return volatility—and volatility may be caused by the same latent variables that cause trading volume.

There is unlikely to be a perfect solution to these problems—partly because existing theory does not give much guidance on the cross-sectional determinants of volume and volatility. Nevertheless, the theory does suggest some variables that might be useful as controls and instruments. A broad spectrum of models [e.g., Copeland (1976), Kyle (1985), Glosten and Milgrom (1985), Wang (1994), He and Wang (1995), and also the dispersion in opinion models of Harris and Raviv (1993), Kandel and Pearson (1995), and Scheinkman and Xiong (2003)] imply that volume ($Turn_{it}$) and the variance of price changes ($Volat_{it}$) should be jointly driven by information flow (I_{it}) and/or liquidity shocks experienced by investors (L_{it}). Andersen (1996), building on Tauchen and Pitts (1983) and Clark (1973), develops an empirical framework that incorporates both information and liquidity shock effects in a time-series setting. Here, I apply these ideas to a cross-sectional setting and augment Eq. (13), which leads to the following structural model for volume and volatility:

$$Turn_{it} = \beta_{0t} + \beta_{11}\tau_{it} + \beta_{12}\tau_{it-1} + \beta_2 I_{it} + \beta_3 L_{it} + \varepsilon_{1it} \quad (14)$$

$$Volat_{it} = \delta_{0t} + \delta_1 I_{it} + \delta_2 L_{it} + \varepsilon_{2it} \ , \quad (15)$$

$$\tau_{it} = \mathrm{g}(Volat_{it}) + \varepsilon_{3\mathrm{it}} \quad (16)$$

The latter equation captures the fact that changes in prices cause changes in characteristics, which cause rule-driven trading volume, but in nonlinear fashion, hence the function g(.). For the following analysis, however, the nature of g(.) is not important. While I_{it} and L_{it} are not observable, one may still be able to identify β_{11} and β_{12} in (14) by using $Volat_{it}$ to capture the effect of the unobservables. Solving Eq. (15) for I_{it} and plugging the result into Eq. (14) yields:

$$Turn_{it} = \alpha_0 + \beta_{11}\tau_{it} + \beta_{12}\tau_{it-1} + \frac{\beta_2}{\delta_1} Volat_{it} + \left[\left(\beta_3 - \frac{\beta_2\delta_2}{\delta_1}\right)L_{it} - \frac{\beta_2}{\delta_1}\varepsilon_{2it} + \varepsilon_{1it}\right], \quad (16)$$

which can be rewritten as

$$Turn_{it} = \alpha_0 + \beta_{11}\tau_{it} + \beta_{12}\tau_{it-1} + \alpha_2 Volat_{it} + v_{it} \ . \quad (17)$$

Consistent estimation of this equation requires instrumental variables: The disturbance v_{it}, which is equal to the term in brackets in Eq. (16), is correlated with $Volat_{it}$, because $Volat_{it}$ measures I_{it} with error. Furthermore, τ_{it} and τ_{it-1} are then also correlated with ν_{it} through their dependence on $Volat_{it}$ and $Volat_{it-1}$. The instruments need to satisfy two requirements: First, they must be partially correlated with the endogenous variables τ_{it} , τ_{it-1}, and $Volat_{it}$. Second, they should be uncorrelated with ε_{2it} and L_{it} to be uncorrelated with ν_{it}. Given such instruments, the parameters β_{11} and β_{12} in Eq. (17) are identified and can be estimated with two-stage least squares (2SLS).

My choice of instruments is motivated as follows. The reason for the endogeneity problem with τ_{it} is that many of the characteristics that contribute to τ_{it} depend on price or price changes in some form, which opens the door for simultaneous effects of I_{it}, for example, on $Turn_{it}$ and τ_{it}. To generate an instrument that is plausibly exogenous, I recalculate τ_{it} excluding all price-related variables (*Ret12*, *Mav12*, *Ret36, S/M*, *L/M*, *DY*, *Beta*, *Volat*, *Size*) and *FutGr*. I denote this instrument τ^*_{it}, and its one-year lag τ^*_{it-1}. Next, to instrument $Volat_{it}$, I draw on evidence in Pástor and Veronesi (2003) that uncertainty about profitability is stronger for no-dividend paying firms with negative earnings. Hence, to the extent that higher uncertainty leads to higher information flow, dummy variables for above-zero earnings and dividends should be correlated with I_{it}. These dummies are valid instruments for $Volat_{it}$ if their errors in measuring I_{it} are not correlated with the measurement error inherent in $Volat_{it}$ (i.e., L_{it} and ε_{2it}). There is no obvious reason why this condition should be violated.

Problems could arise, though, if the true model deviates from the setup in Eqs. (14) to (16). There seem to be two main concerns here. First, $Volat_{it}$ and $Turn_{it}$ might respond differently to private and public information. In models with dispersion in beliefs, public information may cause trading, while in rational expectations models with common priors this is often not the case. Hence, to take an extreme example, suppose that both public and private information cause price changes, but only private information generates trade. In this case, an additional public information flow term would appear in ν_{it} in Eq. (17). As characteristics changes and hence τ^*_{it} might be correlated with public information flow, this would invalidate the instruments. Only if the rate of public information flow is proportional to the rate of private information flow, as assumed in Andersen (1996) in a time-series setting, the public information term disappears from the disturbance and the instruments are valid. In a time-series setting, this assumption seems innocuous. Yet, in a cross-sectional setting, the rate of public to private information flow might vary systematically across firms. Unfortunately, existing theory does not provide much help on this point. A test of the overidentifying restriction (there are four instruments for three endogenous variables) will provide some indication of the severity of this problem.

The second issue is unobserved heterogeneity. The level of trading volume may depend on other omitted exogenous variables that vary across firms, and which could be correlated with τ_{it}. I address this point in two ways. First, I include $Size_{it}$ (average of beginning and end of period log of market capitalization) and $Price_{it}$ (average of beginning and end of period log of price) as control variables. For example, trading volume of larger firms is more likely to be affected by index arbitrage and program trading. The $Price_{it}$ variable is motivated by the fact that trading tends to be thin for stocks with extremely low prices. Second, since these variables might not capture all relevant sources of heterogeneity, I also run 2SLS regressions with industry-time effects. It seems plausible that some of the unobserved heterogeneity could be in the form of inter-industry differences within each cross-section.

Turning to estimation and inference, it is important to take into account that turnover tends to be correlated across firms. Lo and Wang (2000) and Cremers and Mei (2001) document a factor structure in turnover, and thus there is reason to believe that there is non-trivial cross-sectional correlation among the ε_{it}. In this case, standard pooled panel estimators would overstate the statistical significance. To address this problem, I implement both OLS and 2SLS with cross-sectional regressions, using the Fama-MacBeth (1973) method of obtaining standard errors. The regressions are run quarterly with overlapping annual observation windows. Specifically, in each cross-section $t = 1, \ldots, T$, I estimate the regression Eq. (17). This yields a time-series of T parameter estimate vectors $\hat{\beta}_t$ (for 2SLS, there is also a time-series of first-stage estimates). Time-series means of the $\hat{\beta}_t$ then provide consistent estimates of the regression coefficients, and $(1/T)\text{Var}[\hat{\beta}_t]$, adjusted for autocorrelation as in Newey and West (1987), consistently estimates the covariance of their estimation errors. With this approach, for both OLS and 2SLS, the estimation error, including the effect of cross-sectional correlation, is translated into time-series variation of coefficient estimates. The estimated coefficient error covariance matrices in first and second stage regressions can then be used to conduct inference with *t*-statistics and Wald tests. Similarly, as I explain in more detail below, the overidentifying restriction can be tested by comparing the time-averaged test statistic to its expected value under the null hypothesis.

4.2 Ordinary least squares (OLS) results

For the sake of comparison with existing work on the cross-section of trading volume [e.g., Lo and Wang (2000)] and with the subsequent 2SLS results, I start by reporting the OLS results. It is important to keep in mind, however, that the OLS parameter estimates merely show partial relations. They are not estimates of structural parameters. Table 4 presents results for various specifications, with $Turn_{it}$ as the dependent variable and equally weighted observations. To ease the interpretation of magnitudes, the control variables (*Volat*, *Size*, *Price*, etc.) are demeaned and standardized by their cross-sectional standard deviation in each quarter. In model OLS.1, τ_{it} and its lag are the only explanatory variable and they receive large coefficient estimates of 1.85 (std.err. 0.22) and 0.88 (std.err. 0.18), respectively. The R^2 is 10%. Regarding the R^2, it is important to note that it does not necessarily reveal much about the fraction of trading volume that is explained by rule-driven trading, because the magnitude of the R^2 also depends on the amount of cross-sectional variation in τ_{it}.. For example, if τ_{it} did not vary at all across firms, the R^2 in these regressions would be zero, even though τ_{it} could still account for a large fraction of the level of total trading volume. Therefore, the key prediction of the calibrated model concerns the magnitude of the structural coefficients on τ_{it} and τ_{it-1} (their sum should be close to 1.0), not the R^2. The fact that the coefficients in OLS.1 are so large has to

Table 4
Relationship between predicted rule-driven trading volume and actual trading volume (turnover) in NYSE/AMEX stocks: OLS cross-sectional regressions.

Dependent variable is the 12-month average of monthly turnover (number of shares traded/number of shares outstanding). It is regressed on predicted rule-driven turnover (τ_{it}) and its one-year lag (τ_{it-1}). Regressions are run quarterly, using overlapping annual windows, on all NYSE/AMEX stocks that have data on the 15 stock characteristics that are required for the computation of τ_{it} and τ_{it-1}. In each cross-section, observations are equal-weighted. Coefficient estimates shown in the table are the time-series averages of the quarterly estimates. Standard errors of these time-series means are used as an estimator for the coefficient standard errors, as in Fama-MacBeth (1973), adjusted for autocorrelation using the Newey-West method with 12 lags. The standard errors are shown in parentheses. The reported adj. R^2 is the time-series average of the cross-sectional adj. R^2, with corresponding standard error in parentheses. The sample period runs from the first quarter of 1985 to the fourth quarter of 2000.

Model	τ_t	τ_{t-1}	$Volat_t$[a]	$Size_t$[a]	$Price_t$[a]	$\lvert Ret12_t \rvert$[a]	*Forecast Dispersion*$_t$[a]	adj. R^2
OLS.1	1.85	0.88						0.10
	(0.22)	(0.18)						(0.02)
OLS.2					0.28			0.16
					(0.06)			(0.03)
OLS.3	1.12	0.35			0.23			0.19
	(0.07)	(0.11)			(0.06)			(0.04)
OLS.4	1.46	0.89	0.07	0.16	0.35			0.30
	(0.08)	(0.13)	(0.01)	(0.03)	(0.07)			(0.01)
OLS.5	1.83	1.23	0.13	0.01		0.11		0.19
	(0.22)	(0.20)	(0.01)	(0.01)		(0.01)		(0.02)
OLS.6	1.27	0.81	0.04	0.19	0.46		0.04	0.33
	(0.10)	(0.10)	(0.01)	(0.03)	(0.10)		(0.02)	(0.05)

[a] Variables are scaled by their cross-sectional standard deviation.

do with the fact that τ_{it} and τ_{it-1} are positively correlated with $Volat_{it}$, which is strongly related to trading volume, as shown in a large literature surveyed in Karpoff (1987) and Lo and Wang (2000). Hence, not surprisingly, when $Volat_{it}$ is included among the regressors in model OLS.3, the coefficients on τ_{it} and τ_{it-1} drop substantially to 1.12 (std.err. 0.07) and 0.35 (std.err. 0.11). Compared with OLS.2, where $Volat_{it}$ is the only explanatory variable, the coefficient on $Volat_{it}$ however also drops by about a fifth, too. This suggests that rule-driven trading may be one of the reasons why the widely documented volatility-volume relationship exists: Changes in prices lead to changes in characteristics, which in turn cause rule-bound traders to rebalance their portfolios.

Model OLS.4 shows that including $Size_{it}$ and $Price_{it}$ as control variables strengthens rather than weakens the effect of τ_{it} and $\tau_{it\text{-}1}$ with coefficient estimates of 1.46 (std.err. 0.08) and 0.89 (std.err. 0.13). The regressions show that $Price_{it}$ and $Volat_{it}$ are associated with economically large cross-sectional variations in turnover. For example, the coefficient on $Volat_{it}$ suggests that a one-standard deviation difference in $Volat_{it}$ is associated with about 35% difference in annual turnover. Broadly, these results on the control variable effects are consistent with earlier findings by Lo and Wang (2000). Model OLS.5 replaces $Volat_{it}$ with an alternative measure of volatility, $|Ret12|$, the absolute value of 12-month contemporaneous returns. As can be seen in the table, this volatility measure seems to be less strongly related to trading volume and the coefficients on τ_{it} and $\tau_{it\text{-}1}$ are much larger than in OLS.4. Finally, the specification OLS.6 includes a 12-month average of the Diether, Malloy, and Scherbina (2002) analyst forecast dispersion measure. The idea here is that this analyst forecast dispersion might proxy for differences in opinion among investors, which is a potential source of trading activity. The results show that it is indeed positively related to trading activity, but with an economically small coefficient estimate of 0.04 (std.err. 0.02). Compared with OLS.4, the coefficients estimates on τ_{it} and $\tau_{it\text{-}1}$ drop slightly, but with 1.27 (std.err. 0.10) and 0.81 (std.err. 0.10) they are still large.

Overall, the OLS results show that predicted rule-driven trading volume is strongly positively related to actual trading volume. Of course, the OLS results could be driven partly by simultaneity. The 2SLS results in the next section throw some more light on this issue.

4.3 Two-stage least squares (2SLS) results

Table 5 reports the results when a model similar to OLS.4 is estimated by 2SLS. $Volat_{it}$, τ_{it} and $\tau_{it\text{-}1}$ are treated as endogenous. $Size_{it}$ and $Price_{it}$ are assumed to be (econometrically) exogenous in the sense that they are uncorrelated with the disturbance in Eq. (17). The first stage estimates shown in the table reveal that the endogenous variables are strongly related to the instruments. In particular,

Table 5

Relationship between predicted rule-driven trading volume and actual trading volume (turnover) in NYSE/AMEX stocks: 2SLS cross-sectional regressions.

This table shows the result of 2SLS regressions, using a model similar to (OLS.4) in Table 4, but with $Volat_{it}$, τ_{it}, and $\tau_{it\text{-}1}$ treated as endogenous, and instrumented with dummy variables for non-zero dividends (Div_{it}) and Earnings ($Profit_{it}$), as well as $\tau*_{it}$ and $\tau*_{it\text{-}1}$, which are calculated in the same way as τ_{it}, but excluding all price-related variables. First and second stage regressions are run quarterly using overlapping annual windows. Observations are equal-weighted. First and second stage coefficient estimates shown in the table are the time-series averages of the quarterly regression coefficient estimates. Standard errors of these time-series means are used as an estimator for the coefficient standard errors, as in Fama-MacBeth (1973), adjusted for autocorrelation using the Newey-West method with 12 lags. The standard errors are shown in parentheses. The sample period runs from the first quarter of 1985 to the fourth quarter of 2000.

Model	τ_t	$\tau_{t\text{-}1}$	$Volat_t$[a]	$Size_t$[a]	$Price_t$[a]	$Profit_t$	Div_t	$\tau*_t$	$\tau*_{t\text{-}1}$
First stage estimates:									
$Volat_t$				0.16	-0.47	-0.21	-0.57	0.29	0.63
				(0.03)	(0.05)	(0.04)	(0.04)	(0.24)	(1.00)
τ_t				-0.01	-0.01	-0.01	-0.03	0.57	0.10
				(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	(0.01)
$\tau_{t\text{-}1}$				-0.01	-0.01	0.00	-0.03	0.13	0.58
				(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.03)
				Hausman test (exogeneity): $p = 0.000$					
Second stage estimates :									
2SLS	0.55	0.56	0.70	0.02	0.33				
	(0.11)	(0.21)	(0.12)	(0.01)	(0.04)				
				Hausman test (overidentification): $p = 0.117$					

[a] Variables are scaled by their cross-sectional standard deviation.

$Volat_{it}$ tends to be low for firms that have above-zero profits and dividends, consistent with the findings in Pástor and Veronesi (2003). Not surprisingly, there is also a strong relationship between τ_{it} and $\tau_{it\text{-}1}$, and their equivalents with non-price characteristics, $\tau*_{it}$ and $\tau*_{it\text{-}1}$. The fact that the coefficient estimates in the first stage are all highly significant (untabulated Wald tests also reject the null of jointly zero coefficients at $p = 0.000$ for all first stage equations) shows that the estimation is unlikely to suffer from a weak relationship between instruments and endogenous variables of the kind discussed in Staiger and Stock (1997). Based on these instruments, the table also shows p-values from a

Hausman (1978) type test for endogeneity.[15] The hypothesis of exogeneity is strongly rejected (p = 0.000), reinforcing the concern that OLS may be biased.

As might be expected then, the second-stage estimates of the coefficient on τ_{it} and τ_{it-1} turn out to be much smaller than in the OLS case (i.e., compared with OLS.4). They are now 0.55 (std.err. 0.11) and 0.56 (std.err. 0.21). Hence, OLS might indeed have captured some reverse causality going from trading volume to τ_{it} and τ_{it-1}, resulting in upward biased coefficient estimates. Moreover, the coefficient estimate on volatility (0.70) is now twice as high as it is under OLS. This indicates that some of the effect captured by τ_{it} and τ_{it-1} under OLS might in fact be an information flow effect, which is correctly attributed by 2SLS to the information flow indicator $Volat_{it}$. It is also apparent that the use of 2SLS did not produce a large loss of statistical precision compared with OLS—unlike in many 2SLS applications. This is further testimony to the fact that the endogenous variables are strongly related to the instruments.

The important message from this table is that the magnitude of the coefficients fits well with the prediction of the model. In fact, untabulated tests show that one cannot reject the hypothesis that the sum of the coefficients on τ_{it} and τ_{it-1} is equal to 1.0—the magnitude predicted by the rule-trading model. However, one should not take this test too seriously. Given the highly stylized nature of the model, and the fact that the calibration uses data that covers only a specific subset of the investor population, one would not really expect that it gets the magnitude exactly right. A coefficient of, say, 0.7 or 1.3 would certainly also be in the ballpark of estimates that one might consider economically consistent with the trading rules model. Overall, the result that one percent higher predicted rule-driven turnover leads to approximately one percent higher actual turnover corroborates the calibration

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[15] Specifically, each quarter, I store the residuals from the first stage regressions, and include them in a regression of $Turn_{it}$ on the exogenous variables ($Size_{it}$, $Price_{it}$) and the potentially endogenous variables (τ_{it}, τ_{it-1}, $Volat_{it}$). Under the null hypothesis of exogeneity of τ_{it}, τ_{it-1}, and $Volat_{it}$, the coefficient on these residuals should be zero [see, e.g., Wooldridge (2001, p. 118)]. I compute a Wald statistic in each quarter, and draw inference—analogously to the Fama-MacBeth method—by comparing the time-series means of the test statistic to its expected value under the null.

results. In particular, it supports my earlier estimate that 25% of observed NYSE/AMEX trading volume is caused by trading rules.

Since there are four instruments and only three endogenous variables, the model is overidentified. By testing the overidentifying restriction, one can then test whether the instruments are valid, i.e. whether they remove the endogeneity problem. The test statistic is $N\text{R}^2$ from the regression of the second stage disturbance estimate on all the instruments (including all exogenous variables), where N is the number of observations [see, e.g., Wooldridge (2001, p. 122)]. With one overidentifying restriction, $N\text{R}^2 \sim \chi^2(1)$. Here, I calculate $N_t\text{R}_t^2$ for each cross-section t. The null hypothesis is then given by $\overline{N_t\text{R}_t^2} = 1$, the mean of the $\chi^2(1)$ distribution. As before, I adjust the standard error of the time-series mean for autocorrelation. A one-tailed test yields a p-value of 0.117, indicating that the overidentification restriction cannot be rejected at conventional significance levels. This provides some reassurance that the instruments are valid.[16]

One remaining issue is unobserved heterogeneity. There might be omitted variables that are correlated with τ_{it} and $\tau_{it\text{-}1}$. Consistent with the cross-sectional regression framework, I allow for unobserved industry-time effects by demeaning dependent and explanatory variables, including the instruments, within each quarter and industry groups. This is equivalent to running cross-sectional regressions with industry dummies. I use 48 four-digit SIC industry groups, defined as in Fama and French (1997).

[16] In fact, there is there is reason to believe that this test overrejects in this setting. The asymptotic $\chi^2(1)$ distribution of the test statistic only holds when residuals have zero cross-sectional correlation. But as I have argued before, turnover residuals are likely to be correlated across firms due to the factor structure in turnover. Hoxby and Paserman (1998) present Monte Carlo evidence that the above test tends to overreject severely when residuals are clustered.

Table 6

Relationship between predicted rule-driven trading volume and total trading volume (turnover) in NYSE/AMEX stocks: 2SLS cross-sectional regressions with industry-time fixed effects.

This table shows 2SLS regressions similar to those in Table 5, but with industry-time fixed effects included in the regression specification. First and second stage regressions are run quarterly using overlapping annual windows. Observations are equal-weighted. First and second stage regressions are run quarterly using overlapping annual windows. Observations are equal-weighted. First and second stage coefficient estimates shown in the table are the time-series averages of the quarterly regression coefficient estimates. Standard errors of these time-series means are used as an estimator for the coefficient standard errors, as in Fama-MacBeth (1973), adjusted for autocorrelation using the Newey-West method with 12 lags. The standard errors are shown in parentheses. The sample period runs from the first quarter of 1985 to the fourth quarter of 2000.

Model	τ_t	τ_{t-1}	$Volat_t$[a]	$Size_t$[a]	$Price_t$[a]	$Profit_t$	Div_t	τ^*_t	τ^*_{t-1}
First stage estimates:									
$Volat_t$				0.15	-0.47	-0.20	-0.47	0.24	0.64
				(0.03)	(0.04)	(0.04)	(0.04)	(0.23)	(1.00)
τ_t				-0.01	-0.01	-0.01	-0.02	0.56	0.09
				(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.01)
τ_{t-1}				-0.01	-0.01	0.00	-0.02	0.13	0.57
				(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.03)
				Hausman test (exogeneity): $p = 0.000$					
Second stage estimates :									
2SLS	0.78	0.67	0.64	0.04	0.31				
	(0.09)	(0.16)	(0.12)	(0.02)	(0.05)				
				Hausman test (overidentification): $p = 0.121$					

[a] Variables are scaled by their cross-sectional standard deviation.

Table 6 presents the results. As can be seen, little has changed compared with Table 5. In fact, the coefficient estimates on τ_{it} and τ_{it-1} are higher when industry-time effects are included. They are now 0.78 (std.err. 0.09) and 0.67 (std.err. 0.16). Here, the overidentification restriction is not rejected either ($p = 0.121$). In summary, controlling for unobserved heterogeneity at the industry level does not change the result that the sum of the estimated coefficients is close to one, as predicted by the trading rules model. If anything, the model seems to underestimate the magnitudes of rule-driven trading volume.

Table 7
Relationship between predicted style-driven trading volume and total trading volume (turnover) in NYSE/AMEX stocks: OLS and 2SLS subperiod results

This table shows results for OLS and 2SLS second-stage regressions similar to those in Table 4 and 5, but with the sample broken into two subperiods. Regressions are run quarterly using overlapping annual windows. Observations are equal-weighted. Coefficient estimates shown in the table are the time-series averages of the quarterly regression coefficient estimates. Standard errors of these time-series means are used as an estimator for the coefficient standard errors, as in Fama-MacBeth (1973), adjusted for autocorrelation using the Newey-West method with 12 lags. The reported adj. R^2 is the time-series average of the cross-sectional adj. R^2, with corresponding standard error in parentheses.

Period	τ_t	τ_{t-1}	$Size_t$	$Price_t$	$Volat_t$	adj. R^2
Panel A: OLS						
1985 - 1992	1.41	0.52				0.07
	(0.16)	(0.14)				(0.02)
	1.36	0.69	0.09	0.10	0.21	0.20
	(0.11)	(0.12)	(0.02)	(0.01)	(0.02)	(0.02)
1993 - 2000	2.30	1.24				0.14
	(0.14)	(0.08)				(0.01)
	1.56	1.10	0.06	0.23	0.49	0.39
	(0.07)	(0.13)	(0.00)	(0.01)	(0.07)	(0.01)
Panel B: 2SLS						
1985 - 1992	0.49	0.55	0.03	0.25	0.47	
	(0.08)	(0.36)	(0.01)	(0.02)	(0.02)	
1993 - 2000	0.61	0.57	0.00	0.42	0.93	
	(0.19)	(0.19)	(0.01)	(0.02)	(0.13)	

[a] Variables are scaled by their cross-sectional standard deviation.

4.4 Robustness checks

To check the robustness of the results, I have also investigated several variations on the methodology. For example, Table 7 shows OLS and second-stage 2SLS results (as in Table 5) when the sample is broken into two subperiods. As can be seen in the table, the OLS coefficient estimates on τ_{it} and τ_{it-1} are much higher in the second subperiod. In contrast, the sum of the 2SLS coefficients on τ_{it} and τ_{it-1} does not vary much across the two periods (1.04 vs. 1.18). The overidentifying restrictions are not rejected (untabulated p-values are p = 0.606 in the first and p = 0.055 in the second subperiod).

The difference in OLS and 2SLS results appears to be due to stronger association of volatility and volume in the second subperiod. Since 2SLS attributes more of the cross-sectional variation in $Turn_{it}$ to $Volat_{it}$ than OLS does, this might explain the results.

The similarity of the 2SLS results across subperiods may appear surprising. Given that mutual funds hold a much larger fraction of the total market in the second subperiod (based on the portfolio holdings data, their share of total market equity of the NYSE/AMEX stocks my the sample has increased from about 4% in 1985 to about 17% at the end of 2000), one might have expected the coefficient estimates to differ more. If the prevalence of trading rules among mutual funds and other investors were different, the coefficient estimates on τ_{it} and τ_{it-1} would have to change as mutual funds' share of the market and total trading volume goes up over time. Therefore, a conclusion that one could draw from these results is that the distribution of trading rules among other investor groups is similar to what we observe for mutual funds. In this case, calibration based only on mutual funds data would not introduce an upward or downward bias.

Finally, one might wonder whether the inclusion of lagged volatility would impact the results. In most theoretical models, the relationship between volume and volatility is contemporaneous, and thus controlling for contemporaneous volatility, as in Tables 4 to 7, should be sufficient—lagged volatility should not have a causal effect after controlling for contemporaneous volatility. However, there are exceptions. For example, in He and Wang (1995) privately informed investors trade in the absence of new information to unwind the positions that they entered upon receiving news in previous periods. In this case, volume can lag information flow. Untabulated tests show, however, that the inclusion of lagged volatility does not change the basic results. Under OLS, a specification similar to OLS.4, but with one-year lagged volatility included, yields coefficient estimates for τ_{t} and τ_{t-1} of 1.27 (std.err. 0.06) and 0.56 (std.err. 0.07), respectively, close to the estimates in Table 4 (1.46 and 0.89). Under 2SLS, using one-year lags of the *Div* and *Profit* dummies as additional instruments, the coefficient estimates τ_t and τ_{t-1} are 0.38 (std.err. 0.16) and 0.32 (std.err. 0.19). These are lower than the estimates in Table 5 (0.55 and 0.56), but still broadly in line with the model predictions. One cannot reject the hypothesis that the sum of the coefficients is equal to one at conventional significance levels. As a caveat, though, there seems to be a problem with instrument validity in these regressions with lagged volatility: the overidentification restrictions are strongly rejected.

5. Summary and Discussion

The main point of this paper is easily summarized: Investors' fixation on trading rules causes a substantial part of trading volume in equity markets. Many investors tend to follow trading rules that are based on observable stock characteristics. When stock characteristics change over time, these investors rebalance their portfolios to keep them consistent with their trading rules. My estimates suggest that about 25% of NYSE/AMEX volume can be traced to rule-driven rebalancing. This estimate might be conservative, because trading rules may not be limited to the relatively slow moving stock characteristics that my analysis is based on. Technical or quantitative trading rules operating at higher frequency could give rise to additional trading volume.[17]

In the introduction, I suggested product differentiation in delegated portfolio management and the use of simple characteristics-based forecasting heuristics as two potential explanations for the prevalence of trading rules. My findings provide some hints that the second point is likely to play a substantial role. First, size and value/growth characteristics—the attributes most frequently used to differentiate among equity fund managers, e.g. by advisory firms like Morningstar—contribute only moderately to rule-driven trading volume. Size, for example, is just not volatile enough to generate much trade. Returns-based trading rules such as momentum, which are not directly related to popular style categories, seem to be more significant generators of volume. Second, my regression results suggest that rule-driven trading does not seem to be limited to mutual funds. The product

[17] Interestingly, Huddart, Lang, and Yetman (2003) find that trading volume increases when a stock breaks out of its previous 52-week trading range. This provides a hint that higher frequency technical trading rules could be at work.

differentiation story may apply to institutional investors, but it does not explain why individuals, for example, would use trading rules.

By estimating the volume of trade caused by trading rules, this paper takes a step towards a better understanding of these trading motives, but it would be interesting to extend the analysis and explore in more detail the origins of trading rules and their evolution over time. The emergence and evolution of trading strategies has been studied in simulated stock markets with artificial agents [see, e.g., LeBaron (2000)], and one could investigate some of these issues empirically in real financial markets. For instance, one might hypothesize that, if trading rules are indeed a manifestation of investors' forecasting models, their prevalence should depend on their past performance. To give a simple example, after a period of strong returns on momentum strategies, momentum investing might become more popular relative to contrarian strategies, and vice versa. In terms of the model developed in this paper, one could study how the distribution of investors' portfolio locations depends on past returns of stocks with different characteristics. To the extent that such predictable shifts in investors trading rules exist, this could also help in assessing the trading volume that is caused by rule-switching—as opposed to the trades studied in this paper, which are aimed at maintaining a portfolio consistent with a fixed rule.

Appendix

A. Market clearing

The aggregate demand density function can be obtained by integrating the within-portfolio distributions $f(c|m)$ across all investors, i.e. as the convolution of $f(c|m)$ and $f_m(m)$:

$$f_c^D(c) = \int_{-\infty}^{\infty} f(c \mid m) f_m(m) \mathrm{d}m \tag{A.1}$$

Market clearing requires that $f_c^D(c) = f_c(c)$. Since the convolution of two Gaussians is also Gaussian, $f_c^D(c)$ is multivariate normal. Therefore, $f_c^D(c) = f_c(c)$ iff means and variances match. The mean of demand is

$$\mathrm{E}^D[c] = \mathrm{E}_m[\mathrm{E}[c|m]] = \mathrm{E}[m], \tag{A.2}$$

and it must equal $\mathrm{E}[c] = 0$, the mean of supply. A standard variance decomposition implies that

$$\mathrm{Var}^D[c] = \mathrm{Var}_m[\mathrm{E}[c|m]] + \mathrm{E}_m[\mathrm{Var}[c|m]] = \Sigma_m + \Omega_c, \tag{A.3}$$

which must equal $\mathrm{Var}[c] = \Sigma_c$. Taken together this leads to the result stated in the main text.

B. Parameters of *f*(*m*, *c*)

Since $f_m(m)$ and $f(c|m)$ are multivariate normal distributions, it follows that the joint distribution $f(m, c) = f(c|m)f_m(m)$ is multivariate normal. Using the fact that $\mathrm{E}[m] = 0$ and $\mathrm{E}[c] = 0$, the rules for conditional expectations under normality [see, e.g., Anderson (2003, ch. 2)] imply that

$$\mathrm{E}[c \mid m] = \mathrm{Cov}[c, m]\Sigma_m^{-1} m \ . \tag{A.4}$$

By definition, $m = \mathrm{E}[c|m]$ and so, using (A.4), we get

$$\mathrm{Cov}[c, m] = \Sigma_m \ . \tag{A.5}$$

Hence, the joint distribution is given by

$$\begin{pmatrix} m \\ c \end{pmatrix} \sim \mathrm{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_m & \Sigma_m \\ \Sigma_m & \Sigma_c \end{pmatrix}\right]. \tag{A.6}$$

The conditional distributions are

$$c|m \sim \mathrm{N}[m, \Omega_c] \text{ with } \Omega_c = \Sigma_c - \Sigma_m, \tag{A.7}$$

as defined earlier, and

$$m|c \sim \mathrm{N}[\Sigma_m \Sigma_c^{-1} c, \Omega_m], \text{ with } \Omega_m = \Sigma_m - \Sigma_m \Sigma_c^{-1} \Sigma_m . \tag{A.8}$$

C. Proof of the proposition

To evaluate the integral in the definition, Eq. (2), I use the fact that

$$\int_{m \in S} f(m \mid c_t) - f(m \mid c_{t+1}) \mathrm{d}m = 1 - 2 \int_{m \in S} f(m \mid c_{t+1}) \mathrm{d}m \ . \tag{A.9}$$

This relationship holds, because $f(m|c_t)$ and $f(m|c_{t+1})$ are symmetric, and identical except for their means. The integral on the RHS is taken over S, that is, over all values of m for which (see Definition on p. 11)

$$f(m \mid c_{t+1}) < f(m \mid c_t) \ . \tag{A.10}$$

Since $f(m \mid c_t)$ and $f(m \mid c_{t+1})$ are both multivariate normal PDFs with covariance Ω_m^{-1}, (A.10) is equivalent to

$$-(m - \mathrm{E}[m \mid c_{t+1}])' \Omega_m^{-1} (m - \mathrm{E}[m \mid c_{t+1}]) < -(m - \mathrm{E}[m \mid c_t])' \Omega_m^{-1} (m - [m \mid c_t]) \ . \tag{A.11}$$

Without loss of generality, one can redefine the means of the two distributions as $\mathrm{E}[m \mid c_t] = -\Delta m$ and $\mathrm{E}[m \mid c_{t+1}] = 0$, where $\Delta m \equiv \mathrm{E}[m \mid c_{t+1}] - \mathrm{E}[m \mid c_t]$. With these redefined means, simplifying yields

$$2m\Omega_m^{-1}\Delta m < -\Delta m \Omega_m^{-1} \Delta m \tag{A.12}$$

Eq. (A.12) defines a halfspace below a hyperplane. Since the random vector m is multivariate normal conditional on c, integration of $f(m \mid c_{t+1})$ over this halfspace is equivalent to integrating the univariate normal PDF $f(x \mid c_{t+1})$, with the scalar $x \equiv 2m\Omega_m^{-1}\Delta m$, over $x < -\Delta m \Omega_m^{-1} \Delta m$. Standardizing x by its standard deviation conditional on c_{t+1}, which equals $2\sqrt{\Delta m \Omega_m^{-1} \Delta m}$, then leads to integration of a standard normal PDF, and hence the result that

$$\int_{m \in S} f\left(m \mid c_{t+1}\right) \mathrm{d}m = \Phi\left(-\tfrac{1}{2}\sqrt{\Delta m \Omega_m^{-1} \Delta m}\right), \tag{A.13}$$

where $\Phi(.)$ denotes the univariate standard normal CDF. Finally, rules on conditional expectations under joint normality imply that $\Delta m = \Sigma_m \Sigma_c^{-1} \Delta c$. Plugged into (A.13), using the result in (A.9) and applying the definition of rule-driven trading volume then yields the expression stated in the proposition.

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